Chapter 1 The force due to gravity

Section 1.1 Newton's law of universal gravitation

Worked example: Try yourself 1.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf so that the centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.	
Thinking	**Working**
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required, and convert values into appropriate units when necessary.	$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$ $m_1 = 7.0\,\text{kg}$ $m_2 = 5.5\,\text{kg}$ $r = 0.60\,\text{m}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{7.0 \times 5.5}{0.60^2}$
Solve the equation.	$F_g = 7.1 \times 10^{-9}\,\text{N}$

Worked example: Try yourself 1.1.2

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the force of gravitational attraction between the Earth and the Moon, given the following data: $m_{Earth} = 6.0 \times 10^{24}\,\text{kg}$ $m_{Moon} = 7.3 \times 10^{22}\,\text{kg}$ $r_{Moon–Earth} = 3.8 \times 10^{8}\,\text{m}$	
Thinking	**Working**
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required.	$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$ $m_1 = 6.0 \times 10^{24}\,\text{kg}$ $m_2 = 7.3 \times 10^{22}\,\text{kg}$ $r = 3.8 \times 10^{8}\,\text{m}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^{8})^2}$
Solve the equation.	$F_g = 2.0 \times 10^{20}\,\text{N}$

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Worked example: Try yourself 1.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Sun and the Earth is approximately 3.6×10^{22} N. Calculate the acceleration of the Earth and the Sun caused by this force. Compare these accelerations by calculating the ratio $\frac{a_{Earth}}{a_{Sun}}$.

Use the following data:

$m_{Earth} = 6.0 \times 10^{24}$ kg

$m_{Sun} = 2.0 \times 10^{30}$ kg

Thinking	Working
Recall the formula for Newton's second law of motion.	$F = ma$
Transpose the equation to make *a* the subject.	$a = \frac{F}{m}$
Substitute values into this equation to find the accelerations of the Earth and the Sun.	$a_{Earth} = \frac{3.6 \times 10^{22}}{6.0 \times 10^{24}} = 6.0 \times 10^{-3}\ \text{m s}^{-2}$ $a_{Sun} = \frac{3.6 \times 10^{22}}{2.0 \times 10^{30}} = 1.8 \times 10^{-8}\ \text{m s}^{-2}$
Compare the two accelerations.	$\frac{a_{Earth}}{a_{Sun}} = \frac{6.0 \times 10^{-3}}{1.8 \times 10^{-8}} = 3.3 \times 10^{5}$ The acceleration of the Earth is 3.3×10^5 times greater than the acceleration of the Sun.

Worked example: Try yourself 1.1.4

GRAVITATIONAL FORCE AND WEIGHT

Compare the weight of a 1.0 kg mass on the Earth's surface calculated using the formulae $F_{weight} = mg$ and $F_g = G\frac{m_1 m_2}{r^2}$.

Use the following dimensions of the Earth where necessary:

$g = 9.80\ \text{m s}^{-2}$

$m_{Earth} = 6.0 \times 10^{24}$ kg

$r_{Earth} = 6.4 \times 10^{6}$ m

Thinking	Working
Apply the weight equation.	$F_{weight} = mg$ $= 1.0 \times 9.80$ $= 9.8$ N (to two significant figures)
Apply Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$ $F_g = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 1.0}{(6.4 \times 10^{6})^2}$ $= 9.77$ N $= 9.8$ N (to two significant figures)
Compare the two values.	The equations give the same result to two significant figures.

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Worked example: Try yourself 1.1.5

CALCULATING APPARENT WEIGHT

A 79.0 kg student rides a lift down from the top floor of an office block to the ground. During the journey the lift accelerates downwards at 2.35 m s^{-2}, before travelling at a constant velocity of 4.08 m s^{-1} and then finally decelerating at 4.70 m s^{-2}.	
a Calculate the apparent weight of the student in the first part of the journey while accelerating downwards at 2.35 m s^{-2}.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 79.0$ kg $a = 2.35$ m s^{-2} down $g = 9.80$ m s^{-2} down
Apply the sign and direction convention for motion in one dimension. Up is treated as positive and down is negative.	$m = 79.0$ kg $a = -2.35$ m s^{-2} $g = -9.80$ m s^{-2}
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times -2.35) - (79.0 \times -9.80)$ $= -185.65 + 774.2$ $= 589$ N
b Calculate the apparent weight of the student in the second part of the journey while travelling at a constant speed of 4.08 m s^{-1}.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 79.0$ kg $a = 0$ m s^{-2} down $g = 9.80$ m s^{-2} down
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0$ kg $a = 0$ m s^{-2} $g = -9.80$ m s^{-2}
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times 0) - (79.0 \times -9.80)$ $= +774.2$ $= 774$ N

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c Calculate the apparent weight of the student in the last part of the journey while travelling downwards and decelerating at 4.70 m s^{-2}.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 79.0$ kg $a = -4.70$ m s^{-2} down $g = 9.80$ m s^{-2} down
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0$ kg $a = 4.70$ m s^{-2} $g = -9.80$ m s^{-2}
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times 4.70) - (79.0 \times -9.80)$ $= 371.3 + 774.2$ $= 1145.5$ N $= 1.1 \times 10^3$ N

1.1 Review

1 The force of attraction between any two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

2 r is the distance between the centres of the two objects.

3 $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.4 \times 10^{23}}{(2.2 \times 10^{11})^2} = 1.8 \times 10^{21}$ N

4 $F_{weight} = m_{Mars} \times a_{Mars}$

$1.8 \times 10^{21} = 6.4 \times 10^{23} \times a_{Mars}$

$1.8 \times 10^{21} \times a_{Mars} = 6.4 \times 10^{23}$

$a_{Mars} = 2.8 \times 10^{-3}$ m s^{-2}

5 **a** Note: 1 million km = 1×10^6 km = 1×10^9 m

$F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 6.4 \times 10^{23}}{(9.3 \times 10^{10})^2}$

$= 3.0 \times 10^{16}$ N

b $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.0 \times 10^{24}}{(15.3 \times 10^{10})^2}$

$= 3.4 \times 10^{22}$ N

c % comparison = $\frac{(3.0 \times 10^{16})}{(3.4 \times 10^{22})} \times 100 = 0.000\,088\%$. The Mars–Earth force was 0.000 088% of the Sun–Earth force.

6 The Moon has a smaller mass than the Earth and therefore experiences a larger acceleration from the same gravitational force.

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7 $a = g = G\frac{M}{r^2}$

$$g = 6.67 \times 10^{-11} \times \frac{3.3 \times 10^{23}}{(2\,500\,000)^2}$$
$$= 3.5\,\text{m}\,\text{s}^{-2}$$

8 $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23} \times 65}{\left(3.4 \times 10^6\right)^2}$$
$$= 240\,\text{N}$$

9 On Earth, weight is the gravitational force acting on an object near the Earth's surface whereas apparent weight is the contact force between the object and the Earth's surface. In many situations, these two forces are equal in magnitude but are in opposite directions. This is because apparent weight is a reaction force to the weight of an object resting on the ground. However, in an elevator accelerating upwards, the apparent weight of an object would be greater than its weight since an additional force would be required to cause the object to accelerate upwards.

10 **a** $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$
$= ma - mg$
$= (50 \times 1.2) - (50 \times -9.80)$
$= 60 + 490$
$F_N = 550\,\text{N}$

The person's apparent weight is 550 N.

b When the person is moving at a constant speed, their apparent weight is equal to their weight.

$F_N = F_{weight} = mg = 50 \times 9.80$
$= 490\,\text{N}$

11 $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$
$= ma - mg$
$= (45.0 \times -3.15) - (45.0 \times -9.80)$
$= -141.75 + 441$
$= 299\,\text{N}$

The child's apparent weight is 299 N.

12 D. Objects in orbit are in free-fall. While in orbit around the Earth, gravity is reduced, but it is still significant in magnitude.

13 D. At this altitude, gravity is reduced and so will be less than $9.80\,\text{N}\,\text{kg}^{-1}$; hence, acceleration is less than $9.80\,\text{m}\,\text{s}^{-2}$. Note: B is not correct, because although the speed of the satellite would be constant, its velocity is not.

14 A. Apparent weightlessness is felt during free-fall, when F_N is zero.

15 B. In order to be geostationary, the satellite must be in a high orbit.

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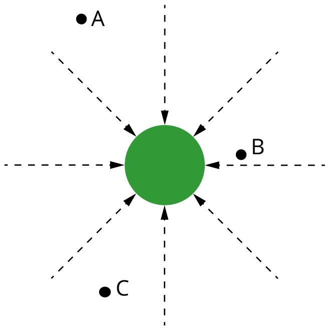
Section 1.2 Gravitational fields

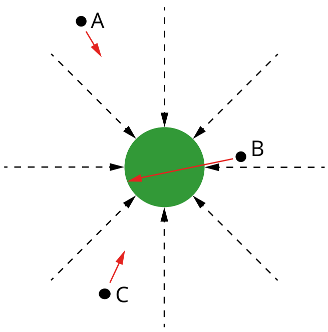
Worked example: Try yourself 1.2.1

INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

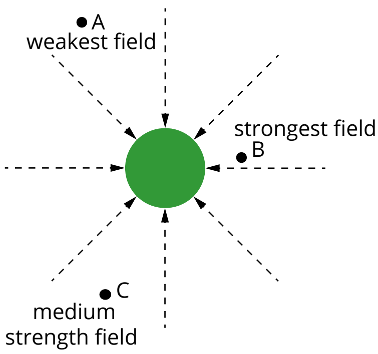
The diagram below shows the gravitational field of a planet.



a Use arrows to indicate the magnitude and direction of the gravitational force acting at points A, B and C.

Thinking	Working
The direction of the field arrows indicates the direction of the gravitational force, which is inwards towards the centre of the planet.	

b Describe the relative strength of the gravitational field at each point.

Thinking	Working
The closer the field lines, the stronger the force.	

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Worked example: Try yourself 1.2.2

CALCULATING GRAVITATIONAL FIELD STRENGTH

A student uses a spring balance to measure the weight of a piece of wood as 2.5 N.
If the piece of wood is thought to have a mass of 260 g, calculate the gravitational field strength indicated by this experiment.

Thinking	Working
Recall the equation for gravitational field strength.	$g = \frac{F_{weight}}{m}$
Substitute in the appropriate values, converting the mass to kg.	$m = 260\text{ g}$ $= 0.26\text{ kg}$ $g = \frac{2.5}{0.26}$
Solve the equation.	$g = 9.6\text{ N kg}^{-1}$

Worked example: Try yourself 1.2.3

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Commercial airlines typically fly at an altitude of 11 000 m. Calculate the gravitational field strength of the Earth at this height using the following data:

$r_{Earth} = 6.38 \times 10^6\text{ m}$

$m_{Earth} = 5.97 \times 10^{24}\text{ kg}$

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Add the altitude to the radius of the Earth.	$r = 6.38 \times 10^6 + 11\,000\text{ m}$ $= 6.391 \times 10^6\text{ m}$
Substitute the values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{(6.391 \times 10^6)^2}$ $= 9.75\text{ N kg}^{-1}$

Worked example: Try yourself 1.2.4

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the strength of the gravitational field on the surface of Mars.

$m_{Mars} = 6.42 \times 10^{23}\text{ kg}$

$r_{Mars} = 3390\text{ km}$

Give your answer correct to two significant figures.

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Convert Mars' radius to m.	$r = 3390\text{ km}$ $= 3.39 \times 10^6\text{ m}$
Substitute values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{(3.39 \times 10^6)^2}$ $= 3.7\text{ N kg}^{-1}$

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1.2 Review

1 N kg^{-1}

2 $g = \frac{F_g}{m} = \frac{1.4}{0.15} = 9.3\text{ N kg}^{-1}$

3 The distance has been increased to three times its original value, from 40 000 km to 120 000 km, so in terms of the inverse square law and the original distance, r:

$$F \propto \frac{1}{r^2}$$
$$\propto \frac{1}{(3r)^2}$$
$$\propto \frac{1}{9r^2}$$

The field strength is $\frac{1}{9}$ of the original value.

4 **a** $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{\left((6380 + 2000) \times 10^3\right)^2}$$
$$= 5.67\text{ N kg}^{-1}$$

b $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{\left((6380 + 10\,000) \times 10^3\right)^2}$$
$$= 1.48\text{ N kg}^{-1}$$

c $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{\left((6380 + 20\,200) \times 10^3\right)^2}$$
$$= 0.56\text{ N kg}^{-1}$$

d $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{\left((6380 + 35\,786) \times 10^3\right)^2}$$
$$= 0.22\text{ N kg}^{-1}$$

5 $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{1 \times 10^{13}}{900^2}$$
$$= 0.0008\text{ N kg}^{-1} \text{ or } 8 \times 10^{-4}\text{ N kg}^{-1}$$

6 $g = G\frac{M}{r^2}$

Mercury: $g = 6.67 \times 10^{-11} \times \frac{3.30 \times 10^{23}}{(2.44 \times 10^6)^2} = 3.7\text{ N kg}^{-1}$

Saturn: $g = 6.67 \times 10^{-11} \times \frac{5.69 \times 10^{26}}{(6.03 \times 10^7)^2} = 10.4\text{ N kg}^{-1}$

Jupiter: $g = 6.67 \times 10^{-11} \times \frac{1.90 \times 10^{27}}{(7.15 \times 10^7)^2} = 24.8\text{ N kg}^{-1}$

7 $g = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{30}}{(10 \times 10^3)^2} = 2 \times 10^{12}\text{ N kg}^{-1}$

8 $g_{\text{poles}} = G\frac{M}{r^2}$

$8.0 = 6.67 \times 10^{-11} \times \frac{M}{5\,000\,000^2}$

$M = 3 \times 10^{24}\text{ kg}$

$g_{\text{equator}} = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3 \times 10^{24}}{6\,000\,000^2} = 5.6\text{ N kg}^{-1}$

$8.0 \div 5.6 = 1.4$

The gravitational field strength at the poles is 1.4 times that at the equator. (Alternatively, the inverse square law could be used to find this relationship.)

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9 Let x be the distance from the centre of Earth to where Earth's gravity equals the Moon's gravity. Then:

$$g_{Earth} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{x^2}$$

$$g_{Moon} = \frac{6.67 \times 10^{-11} \times 7.3 \times 10^{22}}{\left(3.8 \times 10^{8} - x\right)^2}$$

Equating these two expressions gives:

$$\frac{6.0 \times 10^{24}}{x^2} = \frac{7.3 \times 10^{22}}{\left(3.8 \times 10^{8} - x\right)^2}$$

$$\frac{82.2}{x^2} = \frac{1}{\left(3.8 \times 10^{8} - x\right)^2}$$

Taking square roots of both sides gives:

$$\frac{9.07}{x} = \frac{1}{\left(3.8 \times 10^{8} - x\right)}$$

Inverting both sides gives:

$$\frac{x}{9.07} = 3.8 \times 10^{8} - x$$

$x = 3.45 \times 10^{9} - 9.07x$

$10.07x = 3.45 \times 10^{9}$

$x = 3.4 \times 10^{8}\,\text{m}$

10 g is proportional to $\frac{1}{r^2}$, so if g becomes $\frac{1}{100}$ of its value, r must become 10 times its value, so that $\frac{1}{r^2}$ becomes $\frac{1}{100}$.

10 times r means a distance of 10 Earth radii.

Section 1.3 Work in a gravitational field

Worked example: Try yourself 1.3.1

WORK DONE FOR A CHANGE IN GRAVITATIONAL POTENTIAL ENERGY

Calculate the work done (in MJ) to lift a weather satellite of 3.2 tonnes from the Earth's surface to the limit of the atmosphere, which ends at the Karman line (exactly 100 km up from the surface of the Earth). Assume $g = 9.80\,\text{N kg}^{-1}$.	
Thinking	**Working**
Convert the values into the appropriate units.	$m = 3.2$ tonnes $= 3200\,\text{kg}$ $h = 100\,\text{km} = 100 \times 10^3\,\text{m}$
Substitute the values into $E_g = mg\Delta h$. Remember to give your answer in MJ to two significant figures.	$E_g = mg\Delta h$ $= 3200 \times 9.80 \times 100 \times 10^3$ $= 3.136 \times 10^9\,\text{J}$ $= 3.1 \times 10^3\,\text{MJ}$
The work done is equal to the change in gravitational potential energy.	$W = \Delta E = 3.1 \times 10^3\,\text{MJ}$

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Worked example: Try yourself 1.3.2

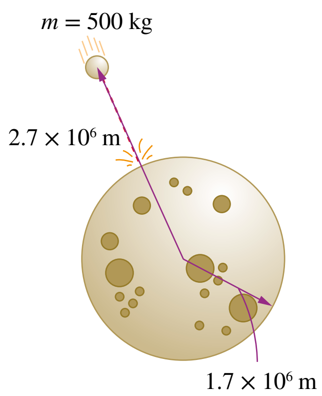
SPEED OF A FALLING OBJECT

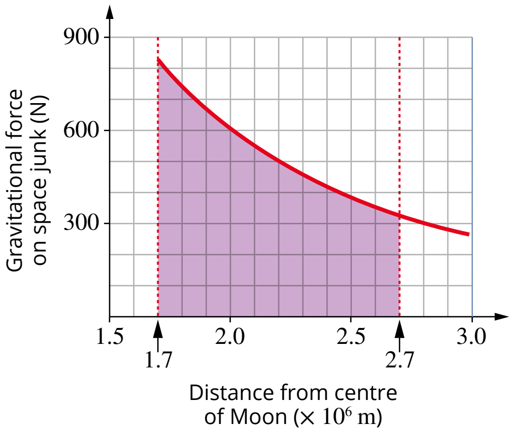
Calculate how fast a 450 g hammer would be going as it hit the ground if it was dropped from a height of 1.4 m on Earth, where $g = 9.80\ \mathrm{N\,kg^{-1}}$.	
Thinking	**Working**
Calculate the gravitational potential energy of the hammer on Earth.	$E_g = mg\Delta h$ $= 0.45 \times 9.80 \times 1.4$ $= 6.2\ \mathrm{J}$
Assume that when the hammer hits the surface of the Earth, all of its gravitational potential energy has been converted into kinetic energy.	$E_k = E_g = 6.2\ \mathrm{J}$
Use the definition of kinetic energy to calculate the speed of the hammer as it hits the ground.	$E_k = \frac{1}{2}mv^2$ $6.2 = \frac{1}{2} \times 0.45 \times v^2$ $\frac{6.2 \times 2}{0.45} = v^2$ $v = 5.2\ \mathrm{m\,s^{-1}}$

Worked example: Try yourself 1.3.3

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE–DISTANCE GRAPH

A 500 kg lump of space junk is plummeting towards the Moon. The Moon has a radius of 1.7×10^6 m. Using the force–distance graph, determine the decrease in gravitational potential energy of the junk as it falls to the Moon's surface.





Thinking	**Working**
Count the number of shaded squares. (Only count squares that are at least 50% shaded.)	Number of shaded squares = 52
Calculate the area (energy value) of each square.	$E_{square} = 0.1 \times 10^6 \times 100$ $= 1 \times 10^7\ \mathrm{J}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square.	$\Delta E_g = 52 \times (1 \times 10^7)$ $= 5.2 \times 10^8\ \mathrm{J}$

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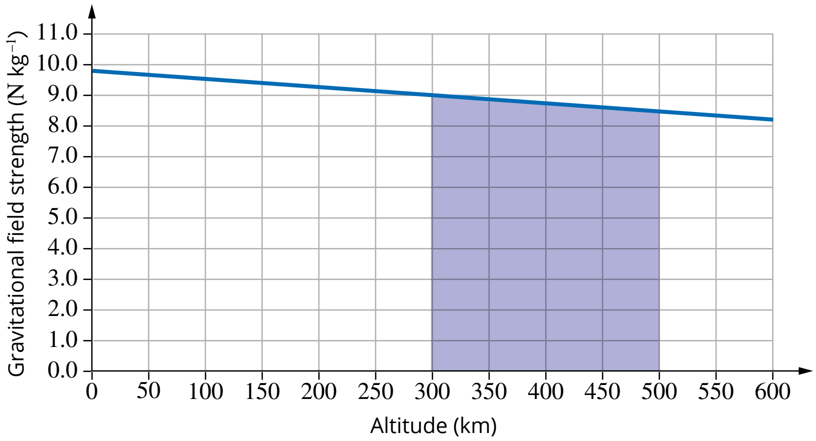
Worked example: Try yourself 1.3.4

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH–DISTANCE GRAPH

A 3000 kg Soyuz rocket moves from an orbital height of 300 km above the Earth's surface to dock with the International Space Station at a height of 500 km. Use the graph of the gravitational field strength of the Earth below to determine the approximate change in gravitational potential energy of the rocket.



Thinking	Working
Count the number of shaded squares. Only count squares that are at least 50% shaded.	Number of shaded squares = 36
Calculate the energy value of each square.	$E_{square} = 50 \times 10^3\,\text{m} \times 1\,\text{N kg}^{-1}$ $= 5 \times 10^4\,\text{J kg}^{-1}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square and the mass of the rocket.	$\Delta E_g = 36 \times 5 \times 10^4 \times 3000$ $= 5.4 \times 10^9\,\text{J}$

1.3 Review

1 C. A stable orbit suggests that the object is in a uniform gravitational field, hence its gravitational potential energy does not change. Its speed will also remain the same in a stable orbit.

2 g increases from point A to point D.

3 The meteor is under the influence of Earth's gravitational field, which will cause it to accelerate at an increasing rate as it approaches the Earth.

4 A, B and C are all correct. The total energy of the system does not change.

5 $W = E_g = 3\,000\,000 \times 9.80 \times 67\,000$
$= 2.0 \times 10^{12}\,\text{J}$

6 $E_g = mg\Delta h$
$= 0.4 \times 6.1 \times 7000$
$= 17\,100\,\text{J}$

$E_k = \frac{1}{2}mv^2$

$17\,100 = \frac{1}{2} \times 0.4 \times v^2$

$v = \sqrt{\frac{2 \times 17\,100}{0.4}}$

$= 292\,\text{m s}^{-1}$

7 a 100 km above the Earth's surface is a distance of 6.4×10^6 m + 100 000 m = 6.5×10^6 m. According to the graph, F is between 9 N and 9.2 N at this height.

b According to the graph, 5 N occurs at approximately 9.0×10^6 m from the centre of the Earth. So, the height above the Earth's surface = $9.0 \times 10^6 - 6.4 \times 10^6 = 2.6 \times 10^6$ m or 2600 km.

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8 **a** Convert $km\,s^{-1}$ to $m\,s^{-1}$ then apply the rule:

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 1 \times 4000^2$$
$$= 8 \times 10^6\,J$$

b $\Delta E_k = \Delta E_g$

ΔE_g = area under the graph

Area = 19 squares × 2 × 0.5 × 10^6

= 1.9×10^7 J

c New E_k = starting $E_k + \Delta E_k = 8 \times 10^6 + 1.9 \times 10^7 = 2.7 \times 10^7$ J

d New speed $= \sqrt{\frac{2 \times 2.7 \times 10^7}{1}}$

$= 7348\,m\,s^{-1}$ or $7.3\,km\,s^{-1}$

9 600 km above the Earth's surface = $6.4 \times 10^6 + 600\,000 = 7.0 \times 10^6$ m or 7000 km

Area under the graph between 7000 km and 8000 km is approximately 7 squares.

As the satellite comes to a stop, the increase in gravitational potential energy over the distance is the same as the E_k at its launch.

The graph is for a 1 kg object, but the satellite is 240 times that mass. So:

$\Delta E_k = \Delta E_g$ = area under the graph × mass of the satellite

= 7 squares × 2 × 0.5 × 10^6 × 240

= 1.7×10^9 J

10 600 km above the Earth's surface = $6.4 \times 10^6 + 600\,000 = 7.0 \times 10^6$ m or 7000 km

2600 km above the Earth's surface = $6.4 \times 10^6 + 2\,600\,000 = 9.0 \times 10^6$ m or 9000 km

The area under the graph between 7000 km and 9000 km is approximately 26 squares.

ΔE_g = area under the graph × mass of the satellite

= $26 \times 1 \times 0.5 \times 10^6 \times 20\,000$

= 2.6×10^{11} J

CHAPTER 1 REVIEW

1 $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 75}{(6.4 \times 10^6)^2}$

= 730 N

2 $F_g = G\frac{m_1 m_2}{r^2}$

$2.79 \times 10^{20} = 6.67 \times 10^{-11} \times \frac{1.05 \times 10^{21} \times 5.69 \times 10^{26}}{r^2}$

$r^2 = \frac{6.67 \times 10^{-11} \times 1.05 \times 10^{21} \times 5.69 \times 10^{26}}{2.79 \times 10^{20}}$

$r = 378\,000\,000$ m

$= 3.78 \times 10^8$ m

3 $F = ma_{Sun}$

$a_{Sun} = \frac{F}{m}$

$= \frac{4.2 \times 10^{23}}{2.0 \times 10^{30}} = 2.1 \times 10^{-7}\,m\,s^{-2}$

4 **a** The force exerted on Jupiter by the Sun is equal in magnitude to the force exerted on the Sun by Jupiter.

b The acceleration of Jupiter caused by the Sun is greater than the acceleration of the Sun caused by Jupiter.

5 $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23}}{(3\,400\,000)^2}$

$= 3.7\,m\,s^{-2}$

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6 **a** $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$

$= ma - mg$

$= (50 \times -0.6) - (50 \times -9.80)$

$= -30 + 490$

$F_N = 460\,N$

The person's apparent weight is 460 N.

b When the person is moving at a constant speed, their apparent weight is equal to their weight: $F_{weight} = F_N = 490\,N$

7 **a** $F = G\frac{m_1 m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1000}{(7.15 \times 10^7)^2}$

$= 2.48 \times 10^4\,N$

b The magnitude of the gravitational force that the comet exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the comet = $2.48 \times 10^4\,N$.

c $a = \frac{F_{net}}{m}$

$= \frac{2.48 \times 10^4}{1000}$

$= 24.8\,m\,s^{-2}$

d $a = \frac{F_{net}}{m}$

$= \frac{2.48 \times 10^4}{1.90 \times 10^{27}}$

$= 1.31 \times 10^{-23}\,m\,s^{-2}$

8 D. At a height of two Earth radii above the Earth's surface, a person is a distance of three Earth radii from the centre of the Earth.

Then $F = \frac{900}{3^2} = \frac{900}{9} = 100\,N$

9 **a** D. $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$

$= ma - mg$

$= (80 \times 30) - (80 \times -9.80)$

$= 2400 + 784$

$F_N = 3184\,N$

The person's apparent weight is 3200 N.

b B. From part (a), the apparent weight is greater than the weight of the astronaut.

c C. Weight is unchanged during lift-off as g is constant.

d A. During orbit, the astronaut is in free-fall.

e D. $F_{weight} = ma$

$= 80 \times 8.2$

$= 656\,N$ or $660\,N$

10 When representing a gravitational field with a field diagram, the direction of the arrowhead indicates the *direction* of the gravitational force and the space between the arrows indicates the *magnitude* of the field. In gravitational fields, the field lines always point towards the source of the field and never cross.

11 $g = \frac{F_{weight}}{m}$

$= \frac{600}{61.5}$

$= 9.76\,N\,kg^{-1}$

12 **a** $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6378 \times 1000)^2}$

$= 9.79\,N\,kg^{-1}$

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b $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6357 \times 1000)^2}$

$= 9.85\ \text{N kg}^{-1}$

$\% = \frac{9.85}{9.79} \times 100 = 100.61\%$

13 **a** $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(2.48 \times 10^7)^2}$

$= 11.1\ \text{N kg}^{-1}$

b C. It will accelerate at a rate given by the gravitational field strength, *g*.

14 $G\frac{M}{(0.8R)^2} = G\frac{m}{(0.2R)^2}$

$\frac{M}{0.64} = \frac{m}{0.04}$

$\frac{M}{m} = \frac{0.64}{0.04} = 16$

15 **a** Increase in E_k = area under the graph between 3×10^6 m and 2.5×10^6 m

$= 6 \text{ squares} \times 10 \times 0.5 \times 10^6 = 3 \times 10^7$ J

b $E_{k(initial)} = \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times 1000^2 = 1 \times 10^7$ J

$E_{k(new)} = 1 \times 10^7 + 3 \times 10^7 = 4 \times 10^7$ J

c $v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 4 \times 10^7}{20}} = 2000\ \text{m s}^{-1}$ or $2\ \text{km s}^{-1}$

d From the graph, $F = 70\ \text{N} = mg$

$g = \frac{70}{20} = 3.5\ \text{N kg}^{-1}$

16 300 km = 300 000 m or 3×10^5 m

From the graph, $g = 9\ \text{N kg}^{-1}$ at this altitude.

17 D. The units for the area under the graph are N m kg^{-1}, which are the same as J kg^{-1}.

18 C. As the satellite falls, its gravitational potential energy decreases. The units on the graph are J kg^{-1}, so therefore C is correct.

19 Increase in E_k = area under the graph × mass of the satellite

$= 35 \text{ squares} \times 1 \times 1 \times 10^5 \times 1000 = 3.5 \times 10^9$ J

20 No. Air resistance will play a major part as the satellite re-enters the Earth's atmosphere.

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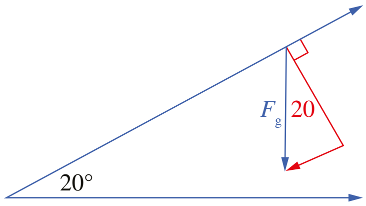
Chapter 2 Motion in a gravitational field

Section 2.1 Inclined planes

Worked example: Try yourself 2.1.1

INCLINED PLANES

A heavier skier of mass 85 kg travels down the same icy slope inclined at 20° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is 9.80 m s^{-2}.

a Determine the components of the weight of the skier perpendicular to the slope and parallel to the slope.	
Thinking	**Working**
Draw a sketch including the values provided.	
Resolve the weight into a component perpendicular to the slope.	The perpendicular component $F_\perp = F_g \cos 20°$ $= 833 \cos 20° = 783$ N
Resolve the weight into a component parallel to the slope.	The parallel component $F_{\|\|} = F_g \sin 20°$ $= 833 \sin 20°$ $= 285$ N
b Determine the normal force that acts on the skier.	
Thinking	**Working**
The normal force is equal in magnitude to the perpendicular component.	$F_N = 783$ N
c Calculate the acceleration of the skier down the slope.	
Thinking	**Working**
Apply Newton's second law. The net force along the incline is the component of the weight parallel to the slope.	$a = \frac{F_{net}}{m}$ $= \frac{285}{85}$ $= 3.35$ $= 3.4$ m s^{-2} down the slope

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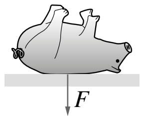
2.1 Review

1 **a**

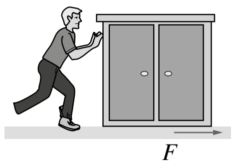
Force exerted on racquet by ball

b



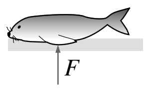
Force exerted on ground by pig

c



Force exerted on ground by wardrobe

d



Gravitational force of attraction that seal exerts on Earth

2 **a** A. The frictional force is opposite to the velocity.

b C. The normal force is perpendicular to the slope.

c $F_{net} = 0$ so $F_f = F_g \sin\theta = 100 \times 9.80 \times \sin 30° = 490\,N$ up the hill.

d Acceleration $a = g\sin\theta = 9.80 \times \sin 30° = 4.90\,m\,s^{-2}$

e Acceleration is not affected by mass if there is no friction.

3 A, B and D. The weight and normal force act on the same body, so they cannot be an action–reaction pair. All other options are true of normal reaction forces.

4 A. $F_N = F_g \cos\theta$, thus the normal force is less than the weight. It is a component of the *weight* that causes the acceleration.

5 **a**
$$\begin{aligned} F_{g\ perp} &= F_g \cos\theta \\ &= mg\cos\theta \\ &= 100 \times 9.80 \times \cos 30° \\ &= 849\,N \end{aligned}$$

b
$$\begin{aligned} F_{g\ parallel} &= F_g \sin\theta \\ &= mg\sin\theta \\ &= 100 \times 9.80 \times \sin 30° \\ &= 490\,N \end{aligned}$$

c $F = ma$

The imbalanced force is parallel to the plane, so using $F_{parallel} = 490\,N$:

$490 = 100 \times a$

$$a = \frac{490}{100}$$

$a = 4.90\,m\,s^{-2}$

6 **a** $F_N = F_g \cos\theta$

Ball 1: $F_N = 0.100 \times 9.80 \times \cos 25° = 0.888\,N$

Ball 2: $F_N = 0.200 \times 9.80 \times \cos 25° = 1.78\,N$

b $a = g\sin\theta$

Ball 1: $a = 9.80 \times \sin 25° = 4.14\,m\,s^{-2}$

Ball 2: $a = 9.80 \times \sin 25° = 4.14\,m\,s^{-2}$

c $F_N = F_g \cos\theta$

Ball 1: $F_N = 0.100 \times 9.80 \times \cos 70° = 0.335\,N$

Ball 2: $F_N = 0.200 \times 9.80 \times \cos 70° = 0.670\,N$

d $a = g\sin\theta$

Ball 1: $a = 9.80 \times \sin 70° = 9.21\,m\,s^{-2}$

Ball 2: $a = 9.80 \times \sin 70° = 9.21\,m\,s^{-2}$

e For (a) the normal force of ball 2 is double that of ball 1. This is because the normal force is directly proportional to mass. For (b) the two balls have the same acceleration, indicative of the fact that acceleration down an inclined plane depends on the angle of the plane, not the mass. Comparing (a) to (c) it can be seen that by increasing the angle of the inclined plane, the normal force acting on the two objects decreases as a function of the angle. You can think of this as the steeper the inclined plane, the closer the object is to free-fall and therefore apparent weightlessness ($F_N = 0$). Finally, comparing (b) to (d) it can be seen that increasing the angle of the inclined plane increases the acceleration of the objects; however, the acceleration of the two objects remains equal.

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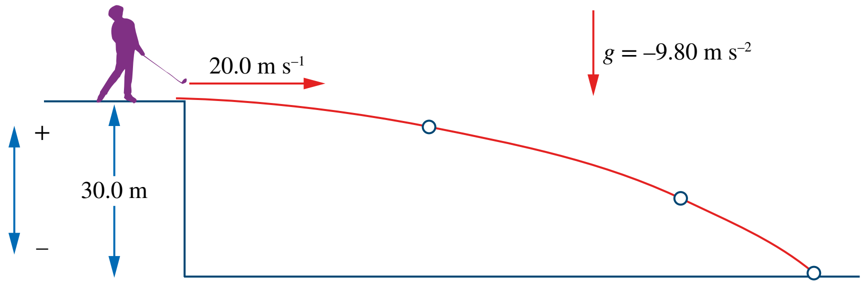
Section 2.2 Projectiles launched horizontally

Worked example: Try yourself 2.2.1

PROJECTILE LAUNCHED HORIZONTALLY

A golf ball of mass 100 g is hit horizontally from the top of a 30.0 m high cliff with a speed of 20.0 m s^{-1}. In your working, use g = 9.80 m s^{-2} and ignore air resistance.



a Calculate the time that the ball takes to land.	
Thinking	**Working**
Let the downwards direction be negative. Write out the information relevant to the vertical component of the motion. Note that the instant the ball is hit, it is travelling only horizontally, so its initial vertical velocity is zero.	Vertically, with down as negative: u = 0 m s^{-1} s = –30.0 m a = –9.80 m s^{-2} t = ?
In the vertical direction, the ball has constant acceleration, so use equations for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$
Substitute values, rearrange and solve for t.	$-30.0 = 0 + (-4.90)t^2$ $t = \sqrt{\frac{-30.0}{-4.90}}$ = 2.47 s
b Calculate the distance that the ball travels from the base of the cliff, i.e. the range of the ball.	
Thinking	**Working**
Write out the information relevant to the horizontal component of the motion. As the ball is hit horizontally, the initial speed gives the horizontal component of the velocity throughout the flight.	Horizontally: u = 20.0 m s^{-1} t = 2.47 s from part (a) s = ?
Select the equation that best fits the information you have.	As horizontal speed is constant, use $v_{av} = \frac{s}{t}$
Substitute values, rearrange and solve for s.	$20.0 = \frac{s}{2.47}$ $s = 20.0 \times 2.47$ = 49.4 m

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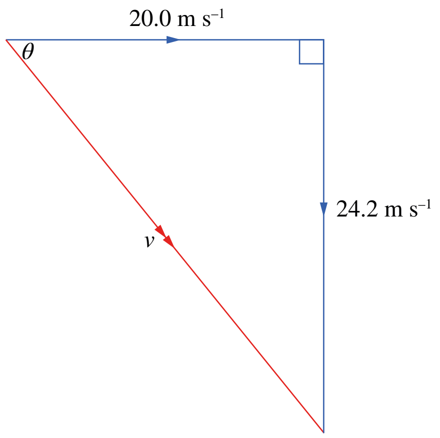
c Calculate the velocity of the ball as it lands.	
Thinking	**Working**
Find the horizontal and vertical components of the ball's speed as it lands. Write out the information relevant to both the vertical and horizontal components.	Horizontally: $u = v = 20.0\ \text{m s}^{-1}$ Vertically, with down as negative: $u = 0$ $a = -9.80\ \text{m s}^{-2}$ $s = -30.0\ \text{m}$ $t = 2.47\ \text{s}$ $v = ?$
To find the final vertical speed, v_v, use the equation for uniform acceleration that best fits the information you have.	Therefore, use $v = u + at$
Substitute values, rearrange and solve for the variable you are looking for, in this case *v*.	$v_v = u + at$ $= 0 + (-9.80) \times 2.47$ $= -24.2$ $= 24.2\ \text{m s}^{-1}$ down
Add the components as vectors.	
Use Pythagoras' theorem to work out the actual speed, *v*, of the ball.	$v = \sqrt{v_h^2 + v_v^2}$ $= \sqrt{(20.0)^2 + (-24.2)^2}$ $= \sqrt{986}$ $= 31.4\ \text{m s}^{-1}$
Use trigonometry to solve for the angle θ.	$\theta = \tan^{-1}\left(\frac{24.2}{20.0}\right)$ $= 50.4°$
Indicate the velocity with magnitude and direction relative to the horizontal.	The final velocity of ball is $31.4\ \text{m s}^{-1}$ at 50.4° below the horizontal.

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Worked example: Try yourself 2.2.2

PROJECTILE LAUNCHED HORIZONTALLY—PROBLEM SOLVING WITH WORK AND ENERGY

A toy plane of mass 300 g is tossed horizontally at 6.00 m s^{-1} from a height of 3.00 m. Using $g = 9.80$ m s^{-2} and ignoring air resistance, calculate the speed of the plane as it lands.

Thinking	Working
Start by calculating the initial kinetic energy of the object, using $E_k = \frac{1}{2}mv_i^2$.	$E_{k\,i} = \frac{1}{2}mv_i^2$ $= \frac{1}{2} \times 0.300 \times 6.00^2$ $= 5.40$ J
Calculate the initial potential energy of the object, using $E_p = mgh_i$.	$E_{p\,i} = mgh_i$ $= 0.300 \times 9.80 \times 3.00$ $= 8.82$ J
Calculate the total mechanical energy.	$E_t = E_{k\,i} + E_{p\,i}$ $= 5.40 + 8.82$ $= 14.22$ J
Finally, use $E_{k\,f} = \frac{1}{2}mv_f^2$ to work out the speed.	$E_t = E_{k\,f} + E_{p\,f}$ $14.22 = \frac{1}{2}mv_f^2 + mgh_f$ $14.22 = \frac{1}{2} \times 0.300 \times v^2 + 0$ $v^2 = 94.8$ $v = 9.74$ m s^{-1}

2.2 Review

1 A and D. Assuming zero air resistance, the only force acting on the stone is gravity, and as a result of gravity, its vertical speed increases. The resultant overall speed, which is a combination of its horizontal and vertical components, will therefore also increase.

2 **a** $v_{av} = \frac{s}{t}$

$2.0 = \frac{s}{0.75}$

$s = 1.5$ m

b Vertically, with down as negative: $u = 0$, $a = -9.80$, $t = 0.75$, $v = ?$

$v = u + at$

$= 0 + (-9.80) \times 0.75$

$= -7.35$ m s^{-1}

c $v = \sqrt{(2.0)^2 + (-7.35)^2}$

$= 7.6$ m s^{-1}

3 **a** The acceleration of a projectile throughout its trajectory is always a constant, $a = 9.80$ m s^{-2} downwards, due to gravity.

b $E_t = \frac{1}{2}mv_i^2 + mgh_i$

$= \frac{1}{2} \times 3.00 \times 4.00^2 + 3.00 \times 9.80 \times 1.20$

$= 59.28$ J

$E_t = \frac{1}{2}mv_f^2 + mgh_f$

$59.28 = \frac{3.00}{2}v^2 + 0$

$v = \sqrt{39.52}$

$v = 6.29$ m s^{-1}

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4 **a** Vertically, with down as negative: $u = 0$, $a = -9.80$, $s = -4.9$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-4.9 = 0 + \frac{1}{2} \times (-9.80) \times t^2$$

$t = 1.0\,\text{s}$

b Horizontally: $u = 20\,\text{m}$, $t = 1$, $s = ?$

$s = v_{av} \times t$

$= 20 \times 1.0$

$= 20\,\text{m}$

c The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity = $9.80\,\text{m}\,\text{s}^{-2}$ down.

d After 0.80 s, the ball has two components of velocity: v_h and v_v

$v_h = 20\,\text{m}\,\text{s}^{-1}$

$v_v = u + at = 0 + (-9.80) \times 0.80 = -7.84\,\text{m}\,\text{s}^{-1}$

The speed of the ball at 0.80 s is given by:

$\sqrt{(20)^2 + (-7.84)^2} = 21.5\,\text{m}\,\text{s}^{-1}$

e The ball will hit the ground 1.0 s after it is struck.

$v_h = 20\,\text{m}\,\text{s}^{-1}$

$v_v = u + at = 0 + (-9.80) \times 1.0 = -9.80\,\text{m}\,\text{s}^{-1}$

The speed of the ball at 1.0 s is given by:

$\sqrt{(20)^2 + (-9.80)^2} = 22.3\,\text{m}\,\text{s}^{-1}$

5 **a** $E_t = \frac{1}{2}mv^2 + mgh_f$

$= \frac{1}{2} \times 2.00 \times 45.0^2 + 2.00 \times 9.80 \times 3.00$

$= 2025 + 58.8$

$= 2083.8\,\text{J}$

$E_t = \frac{1}{2}mv_i^2 + mgh_i$

$2083.8 = \frac{2.00}{2}v^2 + 2.00 \times 9.80 \times 80.0$

$2083.8 = v^2 + 1568$

$v = \sqrt{515.8}$

$v = 22.7\,\text{m}\,\text{s}^{-1}$

b $E_t = \frac{1}{2}mv_i^2 + mgh_i$

$= \frac{1}{2} \times 2.00 \times 45.0^2 + 2.00 \times 9.80 \times 3.00$

$= 2025 + 58.8$

$= 2083.8\,\text{J}$

$E_t = \frac{1}{2}mv^2 + mgh_f$

$2083.8 = \frac{2.00}{2}v^2 + 0$

$v^2 = 2083.8$

$v = \sqrt{2083.8}$

$v = 45.6\,\text{m}\,\text{s}^{-1}$

6 B and C. No atmosphere means no drag, and so the balls travelled in parabolic paths and went much further than they would on Earth.

7 The hockey ball travels further. A polystyrene ball is much lighter and is therefore more strongly affected by air resistance than the hockey ball.

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8 a Vertically with down as negative: $u = 0$, $a = -9.80$, $s = -2.0$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-2.0 = 0 + \frac{1}{2} \times (-9.80) \times t^2$$

$$t = \sqrt{\frac{-2.0}{-4.9}}$$

$$= 0.64\,\text{s}$$

b There is no difference in the time to fall for either ball. Therefore, 0.64 s.

c Ball X:

$$v_{av} = \frac{s}{t}$$

$$5 = \frac{s}{0.64}$$

$s = 3.2\,\text{m}$

Ball Y:

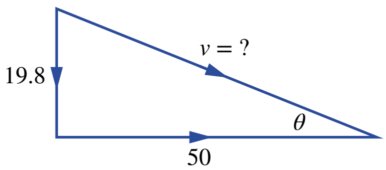
$$v_{av} = \frac{s}{t}$$

$$10 = \frac{s}{0.64}$$

$s = 6.4\,\text{m}$

Difference is $6.4 - 3.2 = 3.2\,\text{m}$

9 a



Vertically, with down as negative: $u = 0$, $a = -9.80$, $s = -20$, $v = ?$

$v^2 = u^2 + 2as$

$= 0 + 2 \times (-9.80) \times (-20)$

$v = \pm\sqrt{392}$

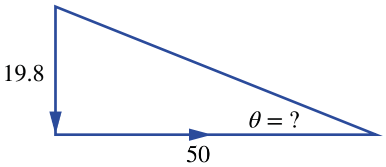
$v = -19.8\,\text{m}\,\text{s}^{-1}$

Horizontally: $u = v = 50\,\text{m}\,\text{s}^{-1}$

$$v = \sqrt{(-19.8)^2 + (50)^2}$$

$$= 54\,\text{m}\,\text{s}^{-1}$$

b



$$\tan\theta = \frac{19.8}{50}$$

$\theta = 22°$

10 a The horizontal velocity of the ball remains constant and $v_h = 10\,\text{m}\,\text{s}^{-1}$ forwards.

b Vertically, with down as negative: $u = 0$, $a = -9.80$, $s = -1.0$, $v = ?$

$v^2 = u^2 + 2as$

$= 0^2 + 2 \times (-9.80) \times (-1.0)$

$v = \pm\sqrt{19.6}$

$v_v = -4.4\,\text{m}\,\text{s}^{-1}$

c $v_v = \sqrt{(-4.4)^2 + (10)^2} = 10.9\,\text{m}\,\text{s}^{-1}$

$$\tan\theta = \frac{4.4}{10}$$

$\theta = 24°$

$v = 10.9\,\text{m}\,\text{s}^{-1}$ at 24° below the horizontal

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d Vertically, with down as negative: $u = 0$, $a = -9.80$, $s = -1.0$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-1.0 = 0 + \frac{1}{2} \times (-9.80) \times t^2$$

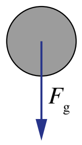
$$t = \sqrt{\frac{-1.0}{-4.90}}$$

$$t = 0.45\,\text{s}$$

e Horizontally: $v = u = 10$, $t = 0.45$, $s = ?$

$$s = v_{av} \times t$$
$$= 10 \times 0.45$$
$$= 4.5\,\text{m}$$

f

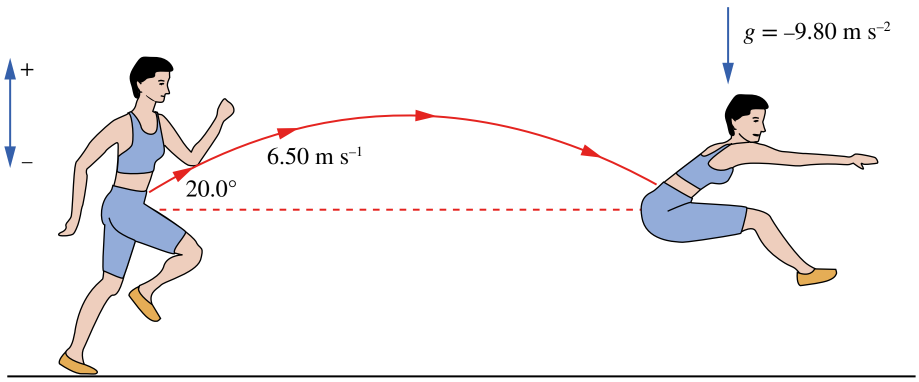


Section 2.3 Projectiles launched obliquely

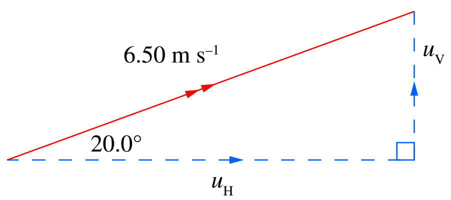
Worked example: Try yourself 2.3.1

LAUNCH AT AN ANGLE

A 50 kg athlete in a long-jump event leaps with a velocity of 6.50 m s^{-1} at 20.0° to the horizontal.



For the following questions, treat the athlete as a point mass, ignore air resistance and use $g = 9.80$ m s^{-2}.

a What is the athlete's velocity at the highest point?	
Thinking	**Working**
First find the horizontal and vertical components of the initial speed.	 Using trigonometry: $u_h = 6.50 \cos 20.0° = 6.11$ m s^{-1} Taking up as positive: $u_v = 6.50 \sin 20.0° = 2.22$ m s^{-1}
Projectiles that are launched obliquely move only horizontally at the highest point. The vertical component of the velocity at this point is therefore zero. The actual velocity is given by the horizontal component of the velocity throughout the motion.	At maximum height $v = 6.11$ m s^{-1} horizontally to the right.

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b What is the maximum height gained by the athlete's centre of mass during the jump?	
Thinking	**Working**
To find the maximum height that is gained, you must work with the vertical component. Recall that at the maximum height, the vertical component of velocity is zero.	Vertically, taking up as positive: $u = 2.22$ $a = -9.80$ $v = 0$ $s = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = 2.22^2 + (-9.80) \times s$
Rearrange and solve for *s*.	$s = \frac{2.22^2}{19.6}$ $= 0.25\,\text{m}$

c Assuming a return to the original height, what is the total time the athlete is in the air?	
Thinking	**Working**
As the motion is symmetrical, the time required to complete the motion will be double that taken to reach the maximum height. First, the time it takes to reach the highest point must be found.	Vertically, taking up as positive: $u = 2.22$ $a = -9.80$ $v = 0$ $t = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v = u + at$ $0 = 2.22 - 9.80 \times t$
Solve for *t* needed to reach maximum height.	$t = \frac{2.22}{9.80}$ $= 0.227\,\text{s}$
The time to complete the motion is double the time it takes to reach the maximum height.	Total time $= 2 \times 0.227$ $= 0.45\,\text{s}$

Worked example: Try yourself 2.3.2

LAUNCH AT AN ANGLE

A little catapult is used to launch a stone of mass 0.250 kg from ground level at a speed of $3.50\,\text{m s}^{-1}$ and an angle of 50.0° above the horizontal. It lands at the same level some distance away.

For the following questions, treat the object as a point mass, ignore air resistance and use $g = 9.80\,\text{m s}^{-2}$.

a What is the stone's velocity at the highest point?	
Thinking	**Working**
First find the horizontal and vertical components of the initial speed.	Using trigonometry: $u_h = 3.50 \cos 50.0°$ $= 2.25\,\text{m s}^{-1}$ $u_v = 3.50 \sin 50.0°$ $= 2.68\,\text{m s}^{-1}$
Projectiles that are launched obliquely move horizontally at the highest point. The vertical component of the velocity at this point is therefore zero. The actual velocity is given by the horizontal component of the velocity throughout the motion.	At maximum height $v = 2.25\,\text{m s}^{-1}$ horizontally to the right.

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b What is the maximum height gained by the stone during the trajectory?	
Thinking	**Working**
To find the maximum height that is gained, you must work with the vertical component. At the maximum height, the stone is moving horizontally and so the vertical component of velocity at this point is zero.	Vertically, taking up as positive: $s = ?$ $u = 2.68\,\text{m s}^{-1}$ $v = 0$ $a = -9.80\,\text{m s}^{-2}$ $t = ?$
Insert these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = 2.68^2 + 2(-9.80)s$
Rearrange the formula and solve for s.	$s = \frac{0 - 2.68^2}{2 \times (-9.80)}$ $s = 0.366\,\text{m}$
c What is the final velocity of the stone as it strikes the ground?	
Thinking	**Working**
Since the stone is landing the same height above the ground as it was launched, it will have the same speed as before.	$v = 3.50\,\text{m s}^{-1}$
By symmetry of the parabola, you know that the velocity vector will just be reflected in the horizontal.	$v = 3.50\,\text{m s}^{-1}$ at 50.0° below the horizontal

2.3 Review

1 B. A javelin travels fastest at launch, then slows on the way up, is *slowest* at the highest point, then speeds up on the way down. At the highest point, the vertical component of the velocity is zero.

2 The optimal launch angle to give the greatest range for any projectile on the ground is 45°.

3 **a** $E_t = \frac{1}{2}mv_i^2 + mgh_i$

$E_t = \frac{1}{2} \times m \times 5.0^2 + m \times 9.80 \times 6.00$

$= 12.5m + 58.8m$

$= 71.3m\,\text{J}$

$E_t = \frac{1}{2}mv_f^2 + mgh_f$

$71.3m = \frac{1}{2}mv^2 + m \times 9.80 \times 3.30$

$71.3 = \frac{1}{2}v^2 + 32.34$

$v = \sqrt{77.92}$

$v = 8.83\,\text{m s}^{-1}$

b $E_t = 71.3m\,\text{J}$

$E_t = \frac{1}{2}mv_f^2 + mgh_f$

$71.3m = \frac{1}{2}m \times (11.1)^2 + m \times 9.80 \times h_f$

$71.3 = \frac{1}{2} \times (11.1)^2 + 9.80 \times h_f$

$h_f = \frac{71.3 - 61.61}{9.80}$

$h_f = 0.989\,\text{m}$

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4 **a** $v_h = v\cos\theta = 15\cos 25° = 13.6\,\text{m s}^{-1}$

b $v_v = v\sin\theta = 15\sin 25° = 6.34\,\text{m s}^{-1}$

c The acceleration is constant and is due to the force of gravity. The acceleration is $9.80\,\text{m s}^{-2}$ down.

d At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight when air resistance is ignored. So the overall velocity at the highest point is equal to the horizontal velocity: $13.6\,\text{m s}^{-1}$.

5 **a** $v_h = v\cos\theta = 8\cos 60° = 4.0\,\text{m s}^{-1}$

b $v_v = v\sin\theta = 8\sin 60° = 6.9\,\text{m s}^{-1}$

c Vertically, with up as positive: $u = 6.9$, $a = -9.80$, $v = 0$, $t = ?$

$v = u + at$

$0 = 6.9 - 9.8t$

$t = 0.70\,\text{s}$

d Vertically, with up as positive: $u = 6.9$, $a = -9.80$, $v = 0$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 6.9^2 + 2 \times (-9.80) \times s$

$6.9^2 = 19.6s$

$s = 2.4\,\text{m}$

Total height = 2.4 + 1.5 = 3.9 m

e The speed is given by the horizontal component of the velocity (as the vertical velocity is zero at this point), so it is $4.0\,\text{m s}^{-1}$.

6 **a** $v_h = 28\cos 30° = 24.2\,\text{m s}^{-1}$ (and remains constant throughout the flight)

b $24.2\,\text{m s}^{-1}$

c $24.2\,\text{m s}^{-1}$

7 Taking up as positive:

a $v_v = 28\sin 30° = 14.0\,\text{m s}^{-1}$

b Vertically: $u = 14$, $a = -9.80$, $t = 1$, $v = ?$

$v = u + at$

$= 14 - 9.80 \times 1.0$

$= 4.20\,\text{m s}^{-1}$

c Vertically: $u = 14$, $a = -9.80$, $t = 2\,\text{s}$, $v = ?$

$v = u + at$

$= 14 - 9.80 \times 2$

$= -5.60 = 5.60\,\text{m s}^{-1}$ down

8 $v = \sqrt{(-5.60)^2 + (24.2)^2}$

$= 24.8\,\text{m s}^{-1}$

9 Vertically with up as positive: $u = 14\,\text{m}$, $a = -9.80$, $v = 0$, $t = ?$

$v = u + at$

$0 = 14 - 9.80t$

$t = 1.43\,\text{s}$

Total time is therefore $2 \times 1.43 = 2.86\,\text{s}$

$v_{av} = \frac{s}{t}$

$24.2 = \frac{s}{2.86}$

$s = 69.2\,\text{m}$

10 C. Air resistance is a force that would act in the opposite direction to the velocity of the ball, thereby producing a horizontal and vertical deceleration of the ball during its flight.

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11 The horizontal component of the initial velocity of the coin is the total speed of the coin at its highest point. Hence:

$$E_t = \frac{1}{2}mv_i^2 + mgh_i$$
$$= \frac{1}{2} \times m \times 7.50^2 + m \times 9.80 \times 10.50$$
$$= 28.125m + 102.9m$$
$$= 131.03m\,\text{J}$$
$$E_t = \frac{1}{2}mv_f^2 + mgh_f$$
$$131.03m = \frac{1}{2}mv^2 + m \times 9.80 \times 0$$
$$131.03 = \frac{1}{2}v^2 + 0$$
$$v = \sqrt{262.06}$$
$$v = 16.2\,\text{m s}^{-1}$$

Section 2.4 Circular motion in a horizontal plane

Worked example: Try yourself 2.4.1

CALCULATING SPEED

A waterwheel has blades 2.0 m in length that rotate at a frequency of 10 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in km h^{-1}.	
Thinking	**Working**
Calculate the period, T. Remember to express frequency in the correct units. Alternatively, recognise that 10 revolutions in 60 seconds means that each revolution takes 6 seconds.	10 revolutions per minute $= \frac{10}{60} = 0.167$ Hz $T = \frac{1}{f}$ $= \frac{1}{0.167} = 6$ s
Substitute r and T into the formula for speed and solve for v.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.0}{6}$ $= 2.09$ m s^{-1}
Convert m s^{-1} into km h^{-1} by multiplying by 3.6.	$2.09 \times 3.6 = 7.5$ km h^{-1}

Worked example: Try yourself 2.4.2

CENTRIPETAL FORCES

An athlete in a hammer throw event is swinging a ball of mass 7.0 kg in a horizontal circular path. The ball is moving at 25 m s^{-1} in a circle of radius 1.2 m.

a Calculate the magnitude of the acceleration of the ball.	
Thinking	**Working**
As the object is moving in a circular path, the centripetal acceleration is towards the centre of the circle. To find the magnitude of this acceleration, write down the other variables that are given.	$v = 25$ m s^{-1} $r = 1.2$ m $a = ?$
Select the equation for centripetal acceleration that fits the information you have, and substitute the values.	$a = \frac{v^2}{r}$ $= \frac{25^2}{1.2}$ $= 521$ m s^{-2}
Calculate the magnitude only, so no direction is needed in the answer.	The acceleration of ball is 521 m s^{-2}.

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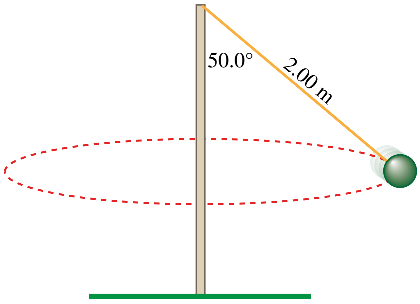
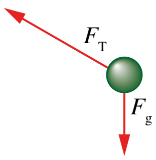
b Calculate the magnitude of the tensile force acting in the wire.	
Thinking	**Working**
Identify the unbalanced force that is causing the object to move in a circular path. Write down the information that you are given.	$m = 7.0\,\text{kg}$ $a = 521\,\text{m}\,\text{s}^{-2}$ $F_{\text{net}} = ?$
Select the equation for centripetal force, and substitute the variables you have.	$F_{\text{net}} = ma$ $= 7.0 \times 521$ $= 3.6 \times 10^3\,\text{N}$
Calculate the magnitude only, so no direction is needed in the answer.	The force of tension in the wire is the unbalanced force that is causing the ball to accelerate. Tensile force $F_T = 3.6 \times 10^3\,\text{N}$

Worked example: Try yourself 2.4.3

OBJECT ON THE END OF A STRING

During a game of totem tennis, the ball of mass 200 g is swinging freely in a horizontal circular path. The cord is 2.00 m long and is at an angle of 50.0° to the vertical, as shown in the diagram.



a Calculate the radius of the ball's circular path.	
Thinking	**Working**
The centre of the circular path is not the top end of the cord, but is where the pole is level with the ball. Use trigonometry to find the radius.	$r = 2.00 \sin 50.0° = 1.53\,\text{m}$
b Draw and identify the forces that are acting on the ball at the instant shown in the diagram.	
Thinking	**Working**
There are two forces acting—the tension in the cord, F_T, and gravity, F_g. These forces are unbalanced.	

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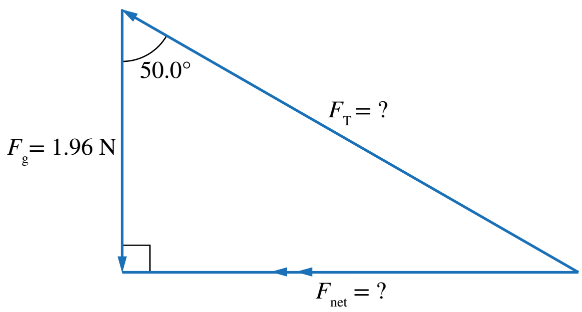
c Determine the net force that is acting on the ball at this time.	
Thinking	**Working**
First calculate the weight force, F_g.	$F_g = mg$ $= 0.200 \times 9.80$ $= 1.96\,N$
The ball has an acceleration that is towards the centre of its circular path. This is horizontal and towards the left at this instant. The net force will also lie in this direction at this instant. A force triangle and trigonometry can be used here.	 $F_{net} = 1.96\tan 50.0° = 2.34\,N$ towards the left
d Calculate the size of the tensile force in the cord.	
Thinking	**Working**
Use trigonometry to find F_T.	$F_T = \frac{1.96}{\cos 50.0°}$ $= 3.05\,N$

2.4 Review

1 B. A sideways force of *friction* between the road and the tyres is enabling the car to travel in a circle.

2 $T = \frac{1}{f}$

$= \frac{1}{5}$

$= 0.2\,s$

3 A and D. The speed is constant, but the velocity is changing, as the direction is constantly changing. The acceleration is directed towards the centre of the circle.

4 **a** $8.0\,m\,s^{-1}$

b $8.0\,m\,s^{-1}$ south

c $a = \frac{v^2}{r} = \frac{(8.0)^2}{9.2} = 7.0\,m\,s^{-2}$ towards the centre, i.e. west

5 $F_{net} = ma = 1200 \times 7.0 = 8.4 \times 10^3\,N$ west

6 **a** $8.0\,m\,s^{-1}$ north

b towards the centre, i.e. east

7 The force needed to give the car a larger centripetal acceleration will eventually exceed the maximum frictional force that could act between the tyres and the road surface. At this time, the car would skid out of its circular path.

8 **a** $a = \frac{v^2}{r}$

$= \frac{(2.0)^2}{1.5}$

$= 2.67\,m\,s^{-2}$

b The skater has an acceleration, so forces are unbalanced.

c $F_{net} = ma = 50 \times 2.7$

$= 135\,N$

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9 **a** $v = 50\,\text{km}\,\text{h}^{-1}$

$= \frac{50}{3.6} = 13.89\,\text{m}\,\text{s}^{-1}$

$= \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2 \times \pi \times 62}{13.89} = 28\,\text{s}$

b $F_{net} = \frac{mv^2}{r}$

$= \frac{1.6 \times 13.89^2}{62}$

$= 5.0\,\text{N}$

10 **a** $T = \frac{1}{f}$

$= \frac{1}{2.0}$

$= 0.5\,\text{s}$

b $v = \frac{2\pi r}{T}$

$= \frac{2 \times \pi \times 0.80}{0.5}$

$= 10\,\text{m}\,\text{s}^{-1}$

c $a = \frac{v^2}{r}$

$= \frac{(10)^2}{0.8}$

$= 125\,\text{m}\,\text{s}^{-2}$

d $F_{net} = ma$

$= 2.5 \times 125$

$= 310\,\text{N}$

11 **a** $r = 2.4\cos 60° = 1.2\,\text{m}$

b The forces are her weight acting vertically and the tension in the rope acting along the rope towards the top of the maypole.

c She has an acceleration directed towards point B, the centre of her circular path.

d Use a force triangle for the girl, showing the net force towards B.

$F_{net} = \frac{mg}{\tan 60°} = \frac{294}{1.73} = 170\,\text{N}$ towards B

e $F_{net} = \frac{mv^2}{r}$

$170 = \frac{30 \times v^2}{1.2}$

$v = 2.6\,\text{m}\,\text{s}^{-1}$

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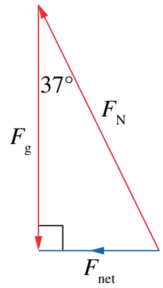
Section 2.5 Circular motion on banked tracks

Worked example: Try yourself 2.5.1

BANKED CORNERS

A curved section of track on an Olympic velodrome has radius of 40 m and is banked at an angle of 37° to the horizontal. A cyclist of mass 80 kg is riding on this section of track at the design speed.

a Calculate the net force acting on the cyclist at this instant as they are riding at the design speed.	
Thinking	**Working**
Draw a force diagram and include all forces acting on the cyclist.	
Calculate the weight force, F_g.	$F_g = mg$ $= 80 \times 9.80$ $= 784\ \text{N}$
Use the force triangle and trigonometry to work out the net force, F_{net}.	$\tan\theta = \frac{F_{net}}{F_g}$ $\tan 37° = \frac{F_{net}}{784}$ $F_{net} = 0.75 \times 784$ $= 590\ \text{N}$
As force is a vector, a direction is needed in the answer.	Net force is 590 N towards the centre of the circle.
b Calculate the design speed for this section of the track.	
Thinking	**Working**
Write down all the known values.	$m = 80\ \text{kg}$ $r = 40\ \text{m}$ $\theta = 37°$ $F_g = 784\ \text{N}$ $F_{net} = 590\ \text{N}$ $v = ?$
Use the design speed formula.	$v = \sqrt{rg\tan\theta}$ $= \sqrt{40 \times 9.80 \times \tan 37°}$ $= 17\ \text{m s}^{-1}$

2.5 Review

1 In all circular motion, the acceleration is directed towards the centre of the circle.

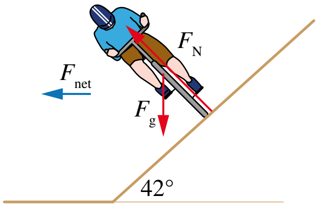
2 The design speed depends on $\tan\theta$ and the radius of the curve, therefore the architect could make the banking angle larger or increase the radius of the track.

3 The car will travel higher up the banked track because the greater speed means that a greater radius is required in the circular path. When the car is travelling faster than the design speed, the normal force is not sufficient to keep it moving in a circle and causes the car to move outwards from the centre.

4 On the horizontal track, the car is depending on the force of **friction** to turn the corner. The **normal** force is equal to the **weight** of the car, so these vertical forces are **balanced**. On the banked track, the **normal** force is not vertical and so is not balanced by the **weight** force. In both cases, the forces acting on the car are unbalanced.

5

C•



6 $v = \sqrt{rg\tan\theta}$

$= \sqrt{28 \times 9.80 \times \tan 33°}$

$= 13.3\ m\ s^{-1}$

$= 13.3 \times 3.6$

$= 48\ km\ h^{-1}$

7 **a** $F_N = \frac{mg}{\cos 33°}$

$= \frac{55 \times 9.80}{\cos 33°}$

$= \frac{539}{\cos 33°}$

$= 642.7 = 640\ N$ (to 2 significant figures)

b On a horizontal track, F_N is equal and opposite to the weight force, so $F_N = mg = 539\ N$. This is less than the normal force on the banked track (643 N).

8 $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{40^2}{150 \times 9.80}\right)$

$= 47°$

9 **a** $F_{net} = \frac{mv^2}{r}$

$= \frac{1200 \times 18^2}{80}$

$= 4860$

$= 4.9\ kN$

b $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{18^2}{80 \times 9.80}\right)$

$= 22°$

10 As the angle of bank (θ) is fixed, an increasing v increases r for constant θ as $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

A greater radius will make the car travel higher up the banked track. The driver would have to turn the front wheels slightly towards the bottom of the bank.

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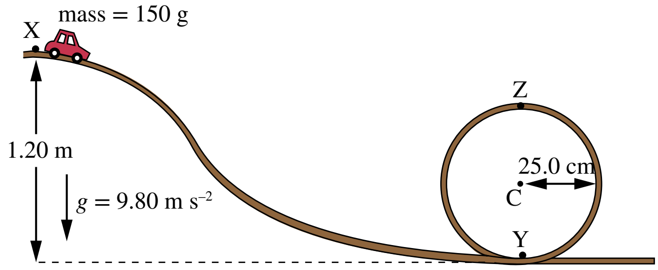
Section 2.6 Circular motion in a vertical plane

Worked example: Try yourself 2.6.1

VERTICAL CIRCULAR MOTION

A student arranges a toy car track with a vertical loop of radius 25.0 cm, as shown.

A toy car of mass 150 g is released from rest at point X at a height of 1.20 m. The car rolls down the track and travels around the loop. Assume $g = 9.80\ \text{m s}^{-2}$, and ignore friction for the following questions.



a Calculate the speed of the car as it reaches the bottom of the loop, point Y.	
Thinking	**Working**
Note all the variables given to you in the question.	At X: $m = 150\ \text{g} = 0.150\ \text{kg}$ $\Delta h = 1.20\ \text{m}$ $v = 0$ $g = 9.80\ \text{m s}^{-2}$
Use an energy approach to calculate the speed. Calculate the total mechanical energy first.	The initial speed is zero, so E_k at X is zero. Mechanical energy, E_m, at X is: $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $= 0 + 0.150 \times 9.80 \times 1.20$ $= 1.76\ \text{J}$
Use conservation of energy ($E_m = E_k + E_g$) to determine the velocity at point Y. As the car rolls down the track, it loses its gravitational potential energy and gains kinetic energy. At the bottom of the loop (Y), the car has zero potential energy.	At Y: $E_m = 1.76\ \text{J}$ $h = 0$ $E_g = 0$ $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = 0.5 \times 0.150v^2 + 0$ $v^2 = 23.5$ $v = \sqrt{23.5}$ $= 4.85\ \text{m s}^{-1}$

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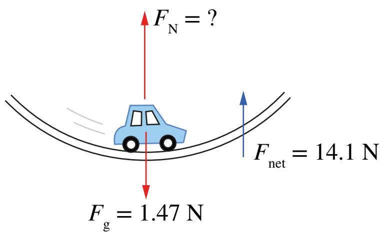
b Calculate the normal reaction force from the track at point Y.

Thinking	Working
To solve for F_N, start by working out the net, or centripetal, force. At Y, the car has a centripetal acceleration towards C (i.e. up), so the net (centripetal) force must also be vertically up at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 4.85^2}{0.250}$ $= 14.1$ N up
Calculate the weight force, F_g, and add it to a force diagram.	At point Y  $F_g = mg$ $= 0.150 \times 9.80$ $= 1.47$ N down
Work out the normal force using vectors. Note up as positive and down as negative for your calculations. These forces are unbalanced, as the car has a centripetal acceleration upwards (towards C). The upwards (normal) force must be larger than the downwards force.	$F_N = 14.1 + 1.47$ $= 15.6$ N up

c What is the speed of the car as it reaches point Z?

Thinking	Working
Calculate the velocity from the values you have, using $E_m = E_k + E_g$.	At Z: $m = 0.150$ kg $r = 25.0$ cm $= 0.250$ m $\Delta h = 2 \times 0.250 = 0.500$ m Mechanical energy is conserved, so use the value from part (a). $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = \frac{1}{2} \times 0.150 \times v^2 + 0.150 \times 9.8 \times 0.500$ $1.76 = 0.075 \times v^2 + 0.735$ $0.075v^2 = 1.76 - 0.735$ $v = \sqrt{13.67}$ $= 3.70$ m s^{-1}

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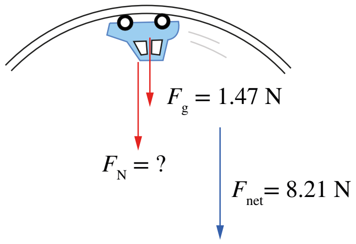
d What is the normal force acting on the car at point Z?	
Thinking	**Working**
To find F_N, start by working out the net, or centripetal, force. At Z, the car has a centripetal acceleration towards C (i.e. down), so the net (centripetal) force must also be vertically down at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 3.70^2}{0.250}$ $= 8.21$ N down
Work out the normal force using vectors. Note up as positive and down as negative for your calculations.	At point Z  $F_{net} = F_g + F_N$ $-8.21 = -1.47 + F_N$ $F_N = -8.21 + 1.47$ $= -6.73$ $= 6.73$ N down

2.6 Review

1 **a** The yo-yo has a constant speed, so its centripetal acceleration $a = \frac{v^2}{r}$ is also constant in magnitude.

b At the bottom of its path, the yo-yo has an upwards acceleration and so the net force is up. This indicates that the tension force is greater than F_g.

c At the top of its path, the yo-yo has a downwards acceleration and so the net force is down. This indicates that the tension force is less than F_g.

d The string is most likely to break at the bottom of its circular path.

2 At this point

$a = \frac{v^2}{r} = g$ so,

$v = \sqrt{rg}$

$= \sqrt{1.5 \times 9.80}$

$= 3.8\,\text{m s}^{-1}$

3 **a** The vertical forces are the weight force from gravity and the normal force from the road.

b $14.4\,\text{km h}^{-1} = 4\,\text{m s}^{-1}$

$F_{net} = \frac{mv^2}{r} = \frac{800 \times 4^2}{10} = 1280\,\text{N}$ (or 1.3×10^3 N)

c Yes. When the car is moving over a hump, the normal force on the driver is less than her weight, mg. Her apparent weight is given by the normal force that is acting and so the driver feels lighter at this point.

d At point of lift-off, $F_N = 0$ and $a = g$.

$a = \frac{v^2}{r} = g$ so,

$v = \sqrt{rg}$

$= \sqrt{10 \times 9.80}$

$= 9.9\,\text{m s}^{-1}$

$= 36\,\text{km h}^{-1}$

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4 **a** At X, mechanical energy is:

$E_m = E_k + E_g$

$= \frac{1}{2}mv^2 + mg\Delta h$

$= 0.5 \times 500 \times 2.00^2 + 500 \times 9.80 \times 50.0$

$= 1000 + 245\,000$

$= 246\,000\,\text{J}$

At Y: E_g is zero so its kinetic energy is 246 000 J

$\frac{1}{2}mv^2 = 246\,000$

$0.5 \times 500 \times v^2 = 246\,000$

$v = \sqrt{984}$

$= 31.4\,\text{m s}^{-1}$

b At Z, mechanical energy = 246 000 J

$E_m = E_k + E_g$

$246\,000 = E_k + 500 \times 9.80 \times 30.0$

$246\,000 = E_k + 147\,000$

$E_k = 99\,000\,\text{J}$

$0.5 \times 500v^2 = 99\,000$

$v = 19.9\,\text{m s}^{-1}$

c At Z: $F_g = mg = 500 \times 9.8 = 4900\,\text{N}$ down

$F_{net} = \frac{mv^2}{r} = \frac{500 \times 19.9^2}{15} = 13\,200\,\text{N}$ down

$F_{net} = F_N + F_g$

$13\,200 = F_N + 4900$

$F_N = 8300\,\text{N}$ down

5 For the cart to just lose contact at Z, $F_N = 0$.

$a = \frac{v^2}{r} = g$ so,

$v = \sqrt{rg}$

$= \sqrt{15.0 \times 9.80}$

$= 12.1\,\text{m s}^{-1}$

6 $F_{net} = F_N + F_g$

$\frac{mv^2}{r} = F_N + 80 \times 9.80$

$\frac{80 \times 35^2}{100} = F_N + 784$

$F_N = 980 - 784 = 196\,\text{N}$ down

7 $F_{net} = \frac{mv^2}{r} = F_N + F_g$, and $F_N = 0$ when losing contact with seat

$\frac{80 \times v^2}{100} = 0 + 80 \times 9.8$

$v^2 = \frac{784 \times 100}{80}$

$= 980$

$v = 31.3\,\text{m s}^{-1}$

Alternatively, use

$a = \frac{v^2}{r} = g$ so,

$v = \sqrt{rg}$

$= \sqrt{100 \times 9.80}$

$= 31.3\,\text{m s}^{-1}$

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8 $a = \frac{v^2}{r}$ so,

$v = \sqrt{rg}$

$= \sqrt{400 \times 9 \times 9.80}$

$= 188\,\text{m s}^{-1}$

9 **a** $a = \frac{v^2}{r}$

$= \frac{6.0^2}{2.0}$

$= 18\,\text{m s}^{-2}$ up

b $F_{net} = \frac{mv^2}{r}$

$= \frac{55 \times 6.0^2}{2.0}$

$= 990\,\text{N}$ up

$F_g = mg$

$= 55 \times 9.8$

$= 540\,\text{N}$ down

$F_{net} = F_N + F_g$ (and taking down as negative)

$990 = F_N - 540$

$F_N = 990 + 540$

$= 1530\,\text{N}$ up

10 **a** If the ball bearing is just losing contact with track, $F_N = 0$ so $F_{net} = F_g$ and therefore $a = 9.80\,\text{m s}^{-2}$ down.

b $v = \sqrt{rg}$

$= \sqrt{0.5 \times 9.80}$

$= 2.2\,\text{m s}^{-1}$

11 **a** $F_{net} = \frac{mv^2}{r}$

$= \frac{500 \times 6.0^2}{5.0}$

$= 3600$

$= 3.6 \times 10^3\,\text{N}$ down

b $F_{net} = F_N + F_g$ (and taking down as negative)

$-3600 = F_N - 4900$

$F_N = -3600 + 4900$

$= 1300\,\text{N}$

$= 1.3 \times 10^3\,\text{N}$ up

c $v = \sqrt{rg}$

$= \sqrt{5.0 \times 9.8}$

$= 7.0\,\text{m s}^{-1}$

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Section 2.7 Satellite motion

Worked example: Try yourself 2.7.1

WORKING WITH KEPLER'S LAWS

Determine the orbital speed of a satellite, assuming it is in a circular orbit of radius of 42 100 km from the centre of Earth. Take the mass of Earth to be 5.97×10^{24} kg and use $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Thinking	Working
Ensure that the variables are in their standard units.	$r = 42\,100 \text{ km} = 4.21 \times 10^7 \text{ m}$
Choose the appropriate relationship between the orbital speed, v, and the data that has been provided.	$a = g = \frac{GM}{r^2} = \frac{v^2}{r}$
Make v, the orbital speed, the subject of the equation.	$v = \sqrt{\frac{GM}{r}}$
Substitute in values and solve for the orbital speed, v.	$v = \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{4.21 \times 10^7}}$ $= 3.08 \times 10^3 \text{ m s}^{-1}$

Worked example: Try yourself 2.7.2

SATELLITES IN ORBIT

Callisto is the second largest of Jupiter's moons. It is about the same size as the planet Mercury. Callisto has a mass of 1.08×10^{23} kg, an orbital radius of 1.88×10^6 km and an orbital period of 1.44×10^6 s (16.7 days).

a Use Kepler's third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.

Thinking	Working
Note down the values for the known satellite. You can work in days and km.	Callisto: $r = 1.88 \times 10^6$ km $T = 16.7$ days
$\frac{r^3}{T^2}$ = constant for all satellites of a central mass. Work out this ratio for the known satellite.	$\frac{r^3}{T^2} = \text{constant}$ $= \frac{(1.88 \times 10^6)^3}{16.7^2}$ $= 2.38 \times 10^{16}$
Use this constant value with the ratio for the satellite in question.	Europa: $r = ?$ $T = 3.55$ days $\frac{r^3}{T^2} = \text{constant}$ $\frac{r^3}{3.55^2} = 2.38 \times 10^{16}$
Make r^3 the subject of the equation.	$r^3 = 3.55^2 \times 2.38 \times 10^{16}$ $= 3.00 \times 10^{17}$
Solve for r.	$r = \sqrt[3]{3.00 \times 10^{17}}$ $= 6.70 \times 10^5$ km Europa has a shorter period than Callisto, so you should expect Europa to have a smaller orbit than Callisto.

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b Use the orbital data for Callisto to calculate the mass of Jupiter.	
Thinking	**Working**
Note down the values for the known satellite. You must work in SI units.	$r_C = 1.88 \times 10^9$ m $T_C = 1.44 \times 10^6$ s $m_C = 1.08 \times 10^{23}$ kg $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $M_J = ?$
Select the expressions from the equation for centripetal acceleration that best suit your data. $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = g$	$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$ These two expressions use the given variables r and T, and the constant G, so that a solution may be found for M.
Transpose to make M the subject.	$M = \frac{4\pi^2 r^3}{GT^2}$
Substitute values and solve.	$M = \frac{4\pi^2 (1.88 \times 10^9)^3}{6.67 \times 10^{-11} \times (1.44 \times 10^6)^2}$ $= 1.90 \times 10^{27}$ kg
c Calculate the orbital speed of Callisto in km s^{-1}.	
Thinking	**Working**
Note values you will need to use in the equation $v = \frac{2\pi r}{T}$.	Callisto: $r = 1.88 \times 10^6$ km $T = 1.44 \times 10^6$ s $v = ?$
Substitute values and solve. The answer will be in km s^{-1} if r is expressed in km.	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.88 \times 10^6}{1.44 \times 10^6}$ $= 8.20$ km s^{-1}

2.7 Review

1 C. Satellites orbit around a central mass. The Earth does not orbit Mars. The Moon does not orbit the Sun and the Sun does not orbit the Earth.

2 D. Increasing the mass of the satellite will not affect its orbital properties.

3 **a** $a = g = 0.22$ m s^{-2}

b $F_g = mg$

$= 2.3 \times 10^3 \times 0.22$

$= 506$ N (or 510 N to two significant figures)

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4 $\frac{r^3}{T^2}$ = constant for satellites of Saturn, therefore the orbital period for each moon can be calculated.

For Atlas:

$$\frac{r^3}{T^2} = \frac{(1.37 \times 10^5)^3}{(0.60)^2}$$
$$= 7.14 \times 10^{15}$$

For Titan:

$$\frac{r^3}{T^2} = 7.14 \times 10^{15}$$
$$T^2 = \frac{r^3}{7.14 \times 10^{15}}$$
$$= \frac{(1.20 \times 10^6)^3}{7.14 \times 10^{15}}$$
$$= 242$$
$$T = \sqrt{242}$$
$$= 15.6 \text{ days}$$

CHAPTER 2 REVIEW

1 B. The ball will increase in speed at a constant rate, that is, with constant acceleration.

2 **a** $a = g\sin\theta$
$= 9.80\sin 30°$
$= 4.9\,\text{m s}^{-2}$

b As $F_N = F_g\cos\theta$, the normal reaction force must be less than the weight force.
$F_N = F_g\cos\theta$
$F_N = F_g\cos 30°$
$F_N = 0.87F_g$

3 **a** $a = g\sin\theta$
$= 9.80\sin 30°$
$= 4.9\,\text{m s}^{-2}$

b $u = 0\,\text{m s}^{-1}$, $s = 2.5\,\text{m}$, $a = 4.9\,\text{m s}^{-2}$, $v = ?$
$v^2 = u^2 + 2as$
$= 0 + 2 \times 4.9 \times 2.5$
$v = \sqrt{24.5}$
$v = 4.9\,\text{m s}^{-1}$

4 **a** $F_N = mg\cos\theta$
$= 57 \times 9.8 \times \cos 65°$
$= 236\,\text{N}$

b $a = g\sin\theta$
$= 9.80\sin 65°$
$= 8.88\,\text{m s}^{-2}$ down the ramp

c $F_{net} = ma = 57 \times 8.88$
$= 506\,\text{N}$ down the ramp

d $u = 0$, $s = 5.0\,\text{m}$, $a = 8.88\,\text{m s}^{-2}$, $v = ?$
$v^2 = u^2 + 2as$
$= 0 + 2 \times 8.88 \times 5.0$
$= 89$
$v = 9.4\,\text{m s}^{-1}$ (speed only)

e $F_{net} = 0$ so forces parallel to incline are balanced.
$F_f = mg\sin\theta = 506\,\text{N}$ up the ramp

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5 **a** Convert speed to m s^{-1}: speed = $\frac{100}{3.6}$ = 27.78 m s^{-1}

Length of the slide: $s = \frac{50}{\sin 70°} = 53.2$ m

To find a, use $v^2 = u^2 + 2as$

$a = \frac{v^2 - u^2}{2s} = \frac{(27.78)^2}{2 \times 53.20} = 7.25$ m s^{-2}

$F_{net} = ma$

$= 70.0 \times 7.25 = 508$ N

b $F_{net} = mg\sin 70° - F_{friction}$

$F_{friction} = 70.0 \times 9.80 \times \sin 70° - 508$

$= 137$ N

c The reaction to the force of the slide on the teenager due to friction is the force of the teenager on the slide.

d The reaction of the weight force on the teenager is the force of gravitational attraction from the teenager on the Earth.

6 **a** $v_{av} = \frac{s}{t}$

$s = v_{av} \times t$

$= 2.5 \times 1$

$= 2.5$ m

b 9.80 m s^{-2} downwards (due to gravity)

7 **a** 10 m s^{-1}. As there are no forces acting horizontally, the horizontal velocity is constant.

b Vertically with down as negative: $u = 0$, $s = -0.97$, $a = -9.80$, $v = ?$

$v^2 = u^2 + 2as$

$= 0 + 2 \times (-9.80) \times (-0.97)$

$v = \pm\sqrt{19.01}$

$v = -4.4$ m s^{-1}

c $v = \sqrt{(10)^2 + (-4.4)^2}$

$= 11$ m s^{-1} (speed only so no direction required)

8 **a** $u_h = 16\cos 50°$

$= 10.3$ m s^{-1}

b $u_v = 16\sin 50°$

$= 12.3$ m s^{-1}

c Vertically with up as positive: $u = 12.3$, $a = -9.80$, $v = 0$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 12.3^2 + 2 \times (-9.80) \times s$

$s = 7.7$ m

Total height from ground is 1.2 + 7.7 = 8.9 m

9 Tait is correct. Energy is conserved at all times. In the absence of air resistance, the trajectory of a projectile will be perfectly symmetrical. Since, when it lands, the ball will have the exact same gravitational potential energy as when it is kicked, it must therefore have the same kinetic energy and hence the same speed. Fred is incorrect because even though the ball will speed up on its journey down, it will have slowed down prior to that, leading to no overall change. Neil is incorrect because Newton's first law states that a body does not need a constant force to keep it moving.

10 **a** $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.1 \times 6.0^2$

$= 1.8$ J

b $E_g = mg\Delta h$

$= 0.1 \times 9.80 \times 2.0$

$= 1.96$ J

c $E_m = 1.8 + 1.96 = 3.76$ J

As it lands $E_m = E_k = \frac{1}{2}mv^2$

$3.76 = \frac{1}{2} \times 0.1 \times v^2$

$v = \sqrt{75.2}$

$v = 8.7$ m s^{-1}

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11 $E_g = mg\Delta h$
$= 0.0400 \times 9.80 \times 1000$
$= 392\,\text{J}$

12 **a** $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.157 \times (20.5)^2$
$= 33.0\,\text{J}$

b $E_g = E_k = 33.0\,\text{J}$

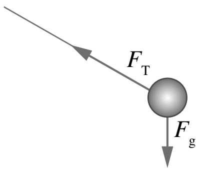
c $E_g = mg\Delta h$
$33.0 = 0.157 \times 9.80 \times \Delta h$
$\Delta h = 21.4\,\text{m}$

13 **a** $v = \frac{2\pi r}{T}$
$= \frac{2\pi \times 0.800}{1.36}$
$= 3.70\,\text{m s}^{-1}$

b $a = \frac{v^2}{r}$
$= \frac{3.70^2}{0.800}$
$= 17.1\,\text{m s}^{-2}$ towards the centre of the circle

c $F_{net} = ma$
$= 0.0250 \times 17.1 = 0.430\,\text{N}$ (size only needed)

14 **a**



b Use a force triangle for the ball.

$F_T = \frac{mg}{\sin 30.0°}$
$= \frac{0.0250 \times 9.80}{0.50}$
$= 0.49\,\text{N}$

15 **a** $a = \frac{v^2}{r}$
$= \frac{5^2}{10}$
$= 2.5\,\text{m s}^{-2}$ towards the centre of the circle

b The centripetal force is created by the friction between the tyres and the ground.

16 **a** $v = \frac{2\pi r}{T}$
$= \frac{2 \times \pi \times 3.84 \times 10^8}{27.3 \times 24 \times 60 \times 60}$
$= 1.02 \times 10^3\,\text{m s}^{-1}$

b $F = \frac{mv^2}{r}$
$= \frac{7.36 \times 10^{22} \times (1.02 \times 10^3)^2}{3.84 \times 10^8}$
$= 1.99 \times 10^{20}\,\text{N}$

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17 Orbital radius $= 6.37 \times 10^6\,\text{m} + 3.6 \times 10^4\,\text{m}$

$= 6.406 \times 10^6\,\text{m}$

Orbital distance $= 2\pi r$

$= 2\pi \times 6.406 \times 10^6\,\text{m}$

$= 4.025 \times 10^7\,\text{m}$

$T = (23 \times 60 \times 60) + (56 \times 60) + 5$

$= 86\,165\,\text{s}$

$v = \frac{2\pi r}{T}$

$= \frac{4.025 \times 10^7}{86165}$

$= 467\,\text{m}\,\text{s}^{-1}$

$a = \frac{v^2}{r}$

$= \frac{(4.67 \times 10^2)^2}{6.406 \times 10^6}$

$= 3.40 \times 10^{-2}\,\text{m}\,\text{s}^{-2}$

18 **a** $10\,\text{m}\,\text{s}^{-1}$ south

b $10\,\text{m}\,\text{s}^{-1}$. Note direction is not required as question asked for speed.

c $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

$= \frac{2 \times \pi \times 20}{10}$

$= 13\,\text{s}$

d $a = \frac{v^2}{r}$

$= \frac{10^2}{20}$

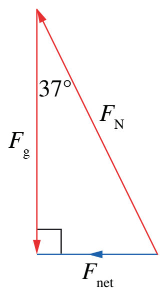
$= 5.0\,\text{m}\,\text{s}^{-2}$ west

e $F_{net} = ma$

$= 1500 \times 5$

$= 7.5 \times 10^3\,\text{N}$ west

19 A. From the triangle, $F_N > F_g$.



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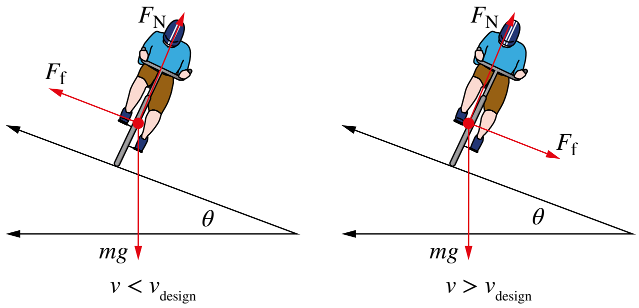
20 a $v = \sqrt{rg\tan\theta}$

$= \sqrt{30.0 \times 9.80 \times \tan 40°}$

$= 15.7\,\text{m s}^{-1}$

b At speeds either above or below the track's design speed there is always sideways frictional force acting on the rider. Liam is travelling slower than the track's design speed, hence there *is* frictional force acting on him. The horizontal component of his normal force is greater than the required centripetal force, hence his bike will be on the verge of sliding down the plane, and the frictional force will be acting up the plane such that the total centripetal force will equal $\frac{mv^2}{r}$.



c $F_{net} = \frac{mv^2}{r}$

$= \frac{90.0 \times 15.7^2}{30.0}$

$= 740\,\text{N}$

$F_{N\ horizontal} = F_N \sin\theta$

With no acceleration in the vertical direction, we have:

$F_N \cos\theta = mg$

$F_N = \frac{mg}{\cos\theta}$

$\therefore F_{N\ horizontal} = \frac{mg}{\cos\theta}\sin\theta$

$= mg\tan\theta$

$= 90.0 \times 9.80 \times \tan 40°$

$= 740\,\text{N}$

This derivation demonstrates how, at the design speed of the track, the net centripetal force is entirely due to the horizontal component of the normal force, and there is no friction contribution. This means that the bike has no tendency to 'want' to move up or down the plane. Note that in our diagram we did not resolve our forces into parallel and perpendicular to the plane. This is because, in a planar cross-section of the track, our cyclist is not moving up and down the plane but rather accelerating horizontally towards the middle. You may also notice that we cannot use $F_N = mg\cos\theta$. This is because there is now a net force towards the middle of the track acting on the cyclist, and hence the perpendicular components of the normal and weight forces cannot be equated.

d If Joe speeds up, he will not necessarily slide off the track. Once his speed is greater than the design speed, there will be friction contributing to his centripetal acceleration. He would need to speed up and overcome this friction before he slid off the track.

21 a i At top:

$F_{net} = \frac{mv^2}{r}$

$= \frac{50 \times 5.0^2}{10}$

$= 125\,\text{N down}$

$F_N = F_g - 125$

$= 490 - 125$

$= 365\,\text{N up}$

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ii At bottom:

$$F_{net} = \frac{mv^2}{r}$$

$$= \frac{50 \times 5.0^2}{10}$$

$$= 125\,\text{N up}$$

$$F_N = F_g + 125$$

$$= 490 + 125$$

$$= 615\,\text{N up}$$

b D. At the top of the ride, $F_N < F_g$, so he would feel lighter than usual.

22 The forces acting on the water when the bucket is directly overhead are the force of gravity (weight) and the normal force from the base of the bucket on the water.

Both of these forces are downwards-acting forces.

The acceleration of the water is towards the centre of the circle, i.e. downwards, and is greater than the acceleration due to gravity.

23 D. Objects in orbit are in free-fall. In Earth's orbit gravity is reduced, but it is still significant in magnitude.

24 D. At this altitude, gravity is reduced and so will be less than $9.80\,\text{N kg}^{-1}$; hence, acceleration is less than $9.80\,\text{m s}^{-2}$. Note: B is not correct, because although the speed of the satellite would be constant, its velocity is not.

25 A. Apparent weightlessness is felt during free-fall, when F_N is zero.

26 $\frac{r^3}{T^2}$ = constant for satellites of Earth, therefore the orbital period for each satellite can be calculated.

For X:

$$\frac{r^3}{T^2} = \text{constant} = k$$

For Y:

$$\frac{(5r)^3}{T_Y^2} = k$$

$$\frac{(5r)^3}{T_Y^2} = \frac{r^3}{T^2}$$

$$\frac{125r^3}{T_Y^2} = \frac{r^3}{T^2}$$

$$T_Y^2 = \frac{125r^3}{r^3}T^2$$

$$= 125T^2$$

$$T_Y = 11.2T$$

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27 **a** $a = \frac{GM}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(3.55 \times 10^8)^2}$

$= 0.054\,\text{m s}^{-2}$

b $a = \frac{v^2}{r}$

$v = \sqrt{ar} = \sqrt{0.054 \times 3.55 \times 10^8}$

$= 4.38 \times 10^3\,\text{m s}^{-1}$

c $F_g = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2} = \frac{GMm}{r^2} = mg$

$\frac{4\pi^2 rm}{T^2} = \frac{GMm}{r^2}$

$T^2 = \frac{4\pi^2 r^3}{GM}$

$= \frac{4\pi^2 (3.55 \times 10^8)^3}{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}$

$= 2.60 \times 10^{11}$

$T = \sqrt{2.60 \times 10^{11}} = 5.09 \times 10^5\,\text{s}$

1 day = 24 × 60 × 60 = 86 400 s

$T = \frac{5.09 \times 10^5}{86400}$

= 5.89 days

28 **a** $a = \frac{GM}{r^2} = g$

$g = \frac{GM}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 7.0 \times 10^{20}}{(3.85 \times 10^5)^2}$

$= 0.315\,\text{N kg}^{-1}$

b $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v = \sqrt{\frac{GM}{r}}$

$= \sqrt{\frac{6.67 \times 10^{-11} \times 7.0 \times 10^{20}}{3.95 \times 10^5}}$

$= 344\,\text{m s}^{-1}$

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Chapter 3 Equilibrium of forces

Section 3.1 Torque

Worked example: Try yourself 3.1.1

CALCULATING TORQUE

A force of 255 N is required to apply a torque on the steering wheel of a sports car as it turns left. The force is applied at 90° to the 15.5 cm radius of the steering wheel. Calculate the torque on the steering wheel.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$\tau = ?$ $r_{\perp} = 0.155\,\text{m}$ $F = 255.0\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_{\perp}F$ $= 0.155 \times 255.0$ $= 39.5\,\text{N m}$
State the answer, including the appropriate direction (clockwise or anticlockwise).	$\tau = 39.5\,\text{N m}$ anticlockwise

Worked example: Try yourself 3.1.2

CALCULATING PERPENDICULAR DISTANCE

A truck driver can apply a maximum force of 1022 N on a large truck wheel-nut spanner that has a length of 80.0 cm. The force is applied at 90° to the radius. If the truck's wheel nuts need a torque of 635 N m to make them secure, determine if the length of this spanner is sufficient for the job.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$\tau = 635\,\text{N m}$ $r_{\perp} = ?$ $F = 1022\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_{\perp}F$ $r_{\perp} = \frac{\tau}{F}$ $= \frac{635}{1022}$ $r_{\perp} = 0.621\,\text{m}$
State the answer with comparable units to the question.	$r_{\perp} = 62.1\,\text{cm}$
Compare the answer with the length of the spanner and state whether it is or isn't appropriate for this task.	As the spanner is 80.0 cm long, it is long enough to provide the minimum perpendicular distance of 62.1 cm.

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Worked example: Try yourself 3.1.3

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.

Calculate the clockwise torque that the mechanic applies to the nut. Give your answer to three significant figures.

Thinking	Working
Use the trigonometric relationship $F_\perp = F\sin\theta$ to determine the force perpendicular to the spanner.	$F_\perp = F\sin\theta$ $= 104.0 \times \sin 75.0°$ $= 100.46\ \text{N}$
Convert variables to their standard units.	$r = 17.0\ \text{cm}$ $= 0.170\ \text{m}$
Apply the equation for torque: $\tau = rF_\perp$	$\tau = rF_\perp$ $= 0.170 \times 100.46$ $= 17.1$
State the answer with the appropriate units including the direction since torque is a vector.	$\tau = 17.1\ \text{N m}$ clockwise

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Worked example: Try yourself 3.1.4

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF DISTANCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.

Source: Dmitry Kalinovsky/Shutterstock.com

Using the perpendicular distance, calculate the clockwise torque that the mechanic applies to the nut. Give your answer to three significant figures.

Thinking	Working
Convert variables to their standard units.	$r = 17.0\text{ cm}$ $= 0.170\text{ m}$
Use the trigonometric relationship $r_\perp = r\sin\theta$ to determine the perpendicular distance from the pivot point to the line of action of the force.	$r_\perp = r\sin\theta$ $= 0.170 \times \sin 75.0°$ $= 0.164\text{ m}$
Apply the equation for torque: $\tau = r_\perp F$	$\tau = r_\perp F$ $= 0.164 \times 104$ $= 17.1$
State the answer with the appropriate unit and direction. Note that this is the same as the answer for Worked example 3.1.3.	$\tau = 17.1\text{ N m}$ clockwise

3.1 Review

1 A. A torque will result. A torque is created when the line of action of the force is not through the pivot point.

2 B. If the force, the force arm, or both increase, then the torque will increase.

3 $\tau = r_\perp F$

$= 0.200 \times 25.0$

$\tau = 5.00\text{ N m}$ in the direction in which the spanner is being turned (as no direction is stated, a description in terms of the action will suffice).

4 $r_\perp = \frac{\tau}{F}$

$r_\perp = \frac{3.47}{12.0}$

$r_\perp = 0.289\text{ m}$

5 D. This situation would provide the greatest torque, as the combination of force and the lever arm length provides a greater torque than the other options.

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6 $\tau = r_{\perp}F$

$= 3.00 \times 5000$

$\tau = 15\,000\ \text{N m}$

$\tau = 1.5 \times 10^{4}\ \text{N m}$

7 $r_{\perp} = \frac{\tau}{F}$

$= \frac{32.1}{24.0}$

$r_{\perp} = 1.34\ \text{m}$

8 $\tau = rF_{\perp}$

$= rF\sin\theta$

$= 2.00 \times 30.0 \times \sin 40°$

$\tau = 38.6\ \text{N m}$

9 $\tau = r_{\perp}F$

$= 0.0700 \times 8.50$

$\tau = 0.595\ \text{N m}$

10 $\theta = 180° - (40° + 90°)$

$= 50.0°$

$\tau = rF\sin\theta$

$= 0.900 \times 82 \times \sin 50°$

$\tau = 56.5\ \text{N m}$

Since the question only asked for the magnitude, there is no need to give a direction of rotation for the applied torque.

11 $r_{\perp} = r\sin\theta$

$= 1.50 \times \sin 30.0°$

$r_{\perp} = 0.750\ \text{m}$

12 $\tau = r_{\perp}F$

$= 0.750 \times 12.5$

$\tau = 9.375 \approx 9.38\ \text{N m}$ clockwise

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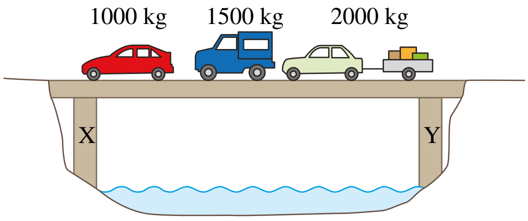
Section 3.2 Equilibrium of forces

Worked example: Try yourself 3.2.1

CALCULATING TRANSLATIONAL EQUILIBRIUM IN ONE DIMENSION

Three cars are parked on a beam bridge that has a mass of 500 kg. Pillar X applies a force of 200 × 10⁴ N upwards. If the situation is in translational equilibrium then calculate the force provided by pillar Y. Use $g = 9.80\ \text{N kg}^{-1}$ when answering this question. Car 1 (C1) is red; car 2 (C2) is blue; car 3 (C3) is light green.



Thinking	Working
Identify the variables involved and state them with their directions in their standard form.	$m_1 = 1000$ kg $m_2 = 1500$ kg $m_3 = 2000$ kg $m_P = 500$ kg $F_X = 2.00 \times 10^4$ N up $g = 9.80\ \text{N kg}^{-1}$ down
Apply a sign convention to the vector data.	$F_X = +2.00 \times 10^4$ N $g = -9.80\ \text{N kg}^{-1}$ down
The object experiencing translational equilibrium is the bridge.	$\Sigma F_{\text{up–down}} = 0$
Expand the equation to include each of the forces acting on the bridge.	$F_{C1} + F_{C2} + F_{C3} + F_B + F_X + F_Y = 0$ $m_{C1}g + m_{C2}g + m_{C3}g + m_Bg + m_Xg + m_Yg = 0$
Substitute the data into the equation and solve for the unknown.	$m_{C1}g + m_{C2}g + m_{C3}g + m_Bg + m_Xg + m_Yg = 0$ $(1000 \times -9.80) + (1500 \times -9.80) + (2000 \times -9.80) + (500 \times -9.80) + 2.00 \times 10^4 + F_Y = 0$ $-9800 + -14\,700 + -19\,600 + -4900 + 2.00 \times 10^4 + F_Y = 0$ $-2.9 \times 10^4 + F_Y = 0$ $F_Y = 2.9 \times 10^4$ N
State the answer with the appropriate direction.	$F_Y = 2.9 \times 10^4$ N up

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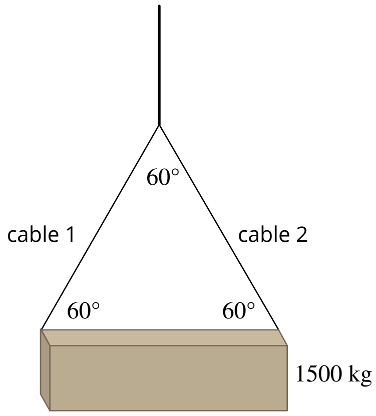
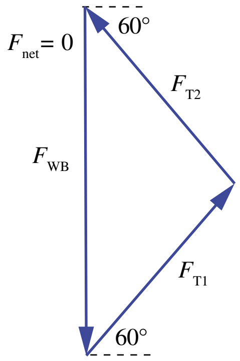
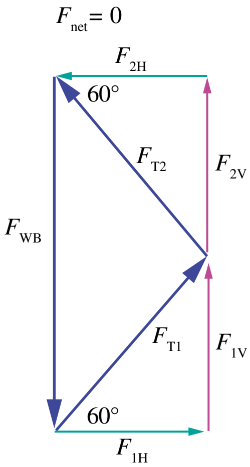
Worked example: Try yourself 3.2.2

CALCULATING TRANSLATIONAL EQUILIBRIUM IN TWO DIMENSIONS

A concrete beam of mass 1500 kg is being lifted by cables labelled 1 and 2, as shown in the diagram. The beam is moving upwards with a constant velocity of 2.0 m s^{-1}. Calculate the tension in cable 1 and cable 2. Ignore the mass of the cables and use g = 9.80 N kg^{-1} when answering this question. Give your answers to three significant figures.



Thinking	Working
Construct a vector diagram adding all of the forces together.	
Apply left and right components and up and down components.	

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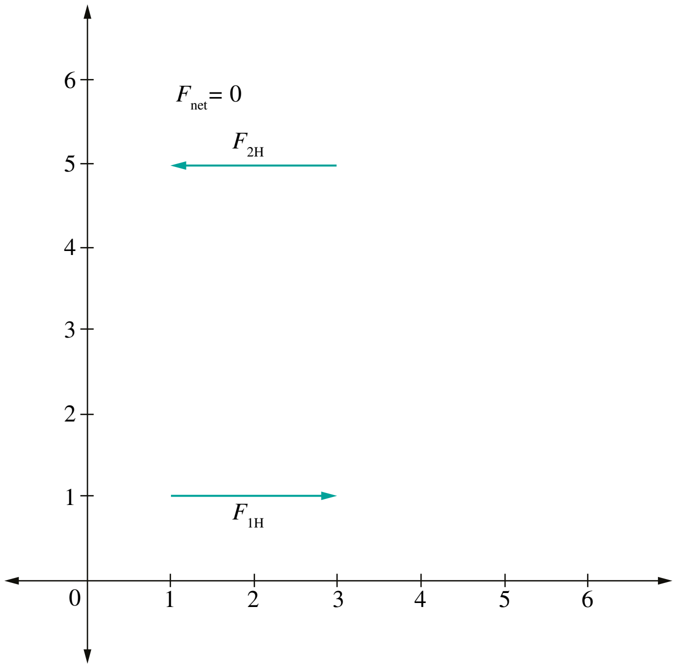
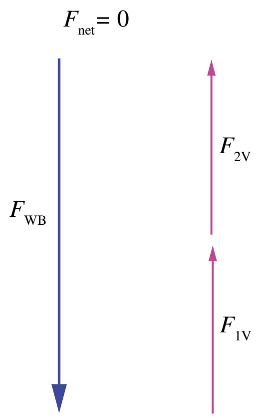
In the horizontal dimension, F_{2H} is in equilibrium with F_{1H}.	$F_{net} = 0$ $F_{2H} = F_{1H}$ 
In the vertical dimension, F_{WB} is in equilibrium with F_{1V} and F_{2V}.	
Apply the equation for translational equilibrium in one dimension. F_{1V} and F_{2V} are equal in magnitude and therefore each is half of F_{WB}.	$\Sigma F_{vertical} = 0$ $F_{1V} = F_{2V}$
Expand the equation to include each of the vertical forces acting on the sign.	$F_{WB} + F_{1V} + F_{2V} = 0$ $m_B g + F_{1V} + F_{2V} = 0$ $(1500 \times -9.80) + F_{1V} + F_{2V} = 0$
Substitute the data into the equation and solve for the unknown.	$-14\,700 + F_{1V} + F_{2V} = 0$ $F_{1V} + F_{2V} = 14\,700$ $F_{1V} = F_{2V} = 7350\text{ N}$

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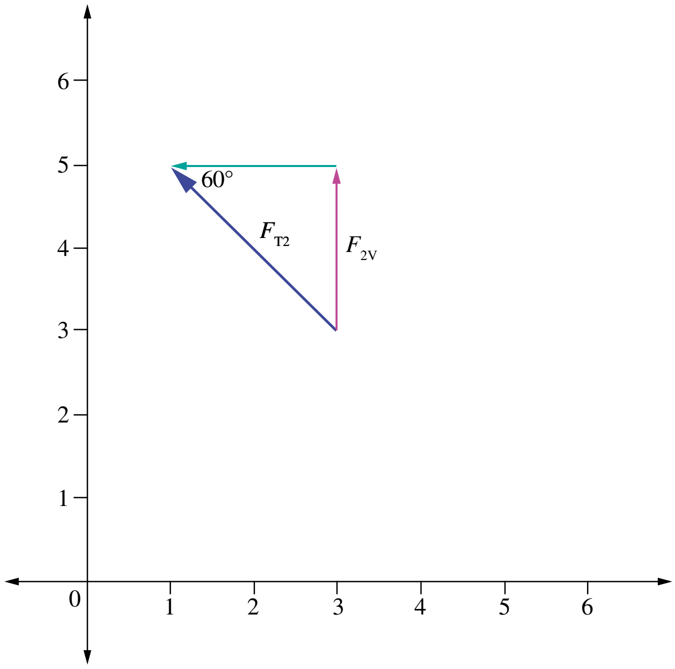
Draw the right triangle with one of the vertical components of the tension and the angle.	
Use trigonometry to solve for the tension in one of the cables, which will equal the tension in the other cable as well.	$\sin\theta = \frac{F_{2V}}{F_{T2}}$ $F_{T2} = \frac{7350}{\sin 60°}$ $= 8490\,N$ $F_{1V} = F_{2V} = 8490\,\text{N}$

3.2 Review

1 D. As there is no net force and therefore no acceleration, the object is in translational equilibrium.

2 D. As the net force is zero, the translational acceleration must also be zero.

3 $\Sigma F = 0$

$F_W + F_T = 0$

$(0.355 \times -9.80) + F_T = 0$

$-3.48 + F_T = 0$

$F_T = 3.48\,\text{N}$

$F_T = 3.48\,\text{N}$ upwards

4 $\Sigma F_y = 0$

$F_W = F_T$

$mg = 7.50$

$m = \frac{7.50}{9.80}$

$m = 0.765\,\text{kg}$

5 $\Sigma F = 0$

$F_{WT1} + F_{WT2} + F_{WFP} + F_{WR} + 2F_T = 0$

$(0.350 \times -9.80) + (0.350 \times -9.80) + (2.25 \times -9.80) + (3.05 \times -9.80) + 2F_T = 0$

$-3.43 + -3.43 + -22.05 + -29.89 + 2F_T = 0$

$-58.80 + 2F_T = 0$

$2F_T = 58.80$

$F_T = \frac{58.80}{2}$

$F_T = 29.4\,\text{N}$

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6 $\Sigma F = 0$

$F_{WT} + F_{WJ} + F_{WP} + 4F_T = 0$

$(79.0 \times -9.80) + (68.0 \times -9.80) + (225 \times -9.80) + 4F_T = 0$

$-774.2 + -666.4 + -2205 + 4F_T = 0$

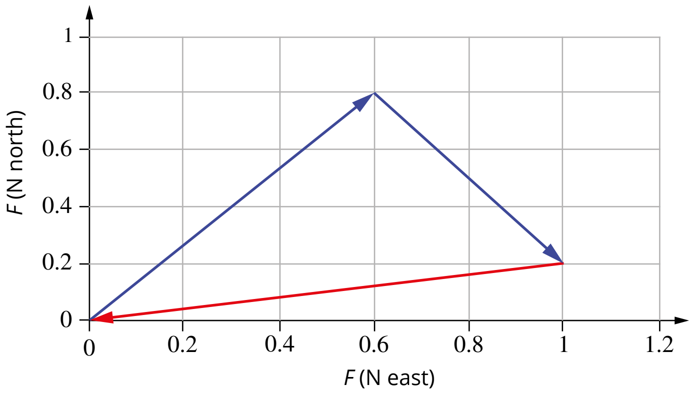
$-3645.6 + 4F_T = 0$

$4F_T = 3645.6$

$F_T = \frac{3645.6}{4}$

$F_T = 911\,N$

7 The force that causes the trolley to be in equilibrium is the force that goes from (1, 0.2) to (0, 0), shown in red.



8 $\Sigma F_y = 0$

$F_{\text{weight bowling ball}} + F_{\text{tension cable A vertical}} = 0$

$(100 \times -9.80) + F_{TAV} = 0$

$F_{TAV} = 980\,N$

$\sin 60° = \frac{F_{TAV}}{F_{\text{tension cable A}}}$

$F_{TA} = \frac{980}{\sin 60°}$

$F_{TA} = 1130\,N$

Cable A = 1130 N or 1.13×10^3 N

$\Sigma F_x = 0$

$F_{\text{tension cable B}} + F_{\text{tension cable A horizontal}} = 0$

$F_{TB} + F_{TA} \cos 60° = 0$

$F_{TB} + 1130 \cos 60° = 0$

$F_{TB} = 565\,N$

Cable B = 565 N

9 $\cos\theta = \frac{F_{LV}}{F_{TL}}$

$F_{LV} = F_{TL} \cos\theta$

$F_{LV} = 40.0 \cos 50°$

$F_{LV} = 25.7\,N = F_{RV}$

$\Sigma F_y = 0$

$F_{WP} + F_{LV} + F_{RV} = 0$

$F_{WP} = 25.7 + 25.7 = 51.4\,N$

$m_P = m_P = \frac{F_{WP}}{g} = \frac{51.4}{9.80}$

$m_P = 5.25\,kg$

m (max) = 5.25 kg

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10 $\Sigma F_y = 0$

$F_{WP} + F_{LV} + F_{RV} = 0$

$(75.0 \times -9.80) + F_{LV} + F_{RV} = 0$

$-735 + F_{LV} + F_{RV} = 0$

$F_{LV} = F_{RV} = \frac{735}{2} = 367.5\,\text{N}$

$\sin\theta = \frac{F_{RV}}{F_{TR}}$

$F_{TR} = \frac{F_{RV}}{\sin\theta}$

$F_{TR} = \frac{367.5}{\sin 10°}$

$F_{TR} = 2116.35 = 2.12 \times 10^3\,\text{N}$

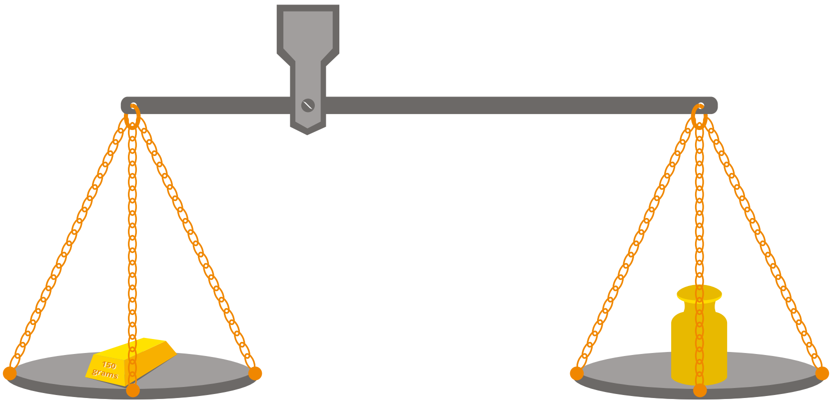
$F_T = 2120\,\text{N}$

Section 3.3 Static equilibrium

Worked example: Try yourself 3.3.1

CALCULATING STATIC EQUILIBRIUM

A set of scales (with one longer arm) is used to measure the mass of gold, when it is larger than the standard set of masses. A lump of gold weighing 150 g is placed on the short arm, which is 10.0 cm long, and the standard masses are placed on the long arm. Use $g = 9.80\,\text{N kg}^{-1}$ in your calculations where required. Give your answers to three significant figures.



a Calculate the force applied to the scale's arm due to the pivot point if a standard mass of 50.0 g exactly balances the gold.	
Thinking	**Working**
Identify the variables involved and state them with their directions in their standard form.	$m_g = 0.150\,\text{kg}$ $m_m = 0.0500\,\text{kg}$ $g = 9.80\,\text{N kg}^{-1}$ down
Apply a sign convention to the vector force data.	$g = -9.80\,\text{N kg}^{-1}$
Identify the object that is in translational equilibrium. This is the object on which all the forces are acting.	The object experiencing translational equilibrium is the scale arm.
Apply the equation for translational equilibrium in one dimension.	$\Sigma F_y = 0$
Expand the equation to include each of the forces acting on the scale arm.	$F_{Wg} + F_{Wm} + F_P = 0$ $m_g g + m_m g + F_P = 0$

Substitute the data into the equation and solve for the unknown.	$(0.150 \times -9.80) + (0.0500 \times -9.80) + F_P = 0$ $-1.47 + (-0.490) + F_P = 0$ $-1.96 + F_P = 0$ $F_P = 1.96\,N$
State the answer with the appropriate direction.	$F_P = 1.96\,N$ up
b Calculate the length the long arm should have in order to balance the gold.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$m_g = 0.150\,kg$ $m_m = 0.0500\,kg$ $r_{\perp g} = 0.100\,m$ $g = 9.80\,N\,kg^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the scale arm.
Decide the reference point about which the torques will be calculated.	The reference point is the pivot of the scale arm.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the standard mass on the scale arm provides the clockwise torque. The force of the gold on the scale arm provides the anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$
Expand the equation to include each of the torques acting on the scale arm.	$F_{Wm}r_{\perp m} = F_{Wg}r_{\perp g}$
Substitute the data into the equation and solve for the unknown.	$F_{Wm}r_{\perp m} = F_{Wg}r_{\perp g}$ $m_m g r_{\perp m} = m_W g r_{\perp g}$ $0.0500 \times 9.80 r_{\perp m} = 0.150 \times 9.80 \times 0.100$ $r_{\perp m} = \frac{0.150 \times 9.80 \times 0.100}{0.0500 \times 9.80}$ $r_{\perp m} = 0.300\,m$
State the answer with comparable units to the question.	The long arm should be 30.0 cm long.

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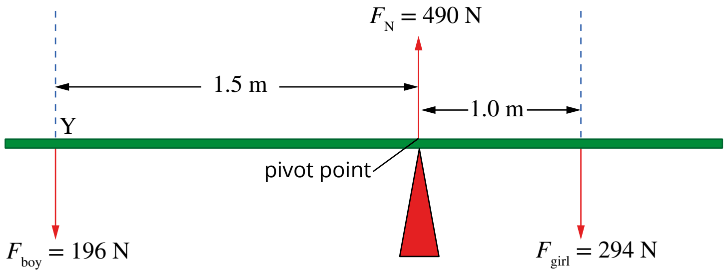
Worked example: Try yourself 3.3.2

CALCULATING STATIC EQUILIBRIUM USING A DIFFERENT REFERENCE POINT

Verify that the seesaw plank in the figure below is also in rotational equilibrium about the reference point Y, where the boy is sitting. The weights of the boy and girl are 196 N and 294 N respectively, and the force of the pivot on the plank is 490 N upwards. Assume that the plank's mass is negligible.



Thinking	Working
Identify the variables involved and state them in their standard form.	$F_P = 490\,N$ $F_g = 294\,N$ $F_b = 196\,N$ $r_{\perp g} = 2.50\,m$ $r_{\perp P} = 1.00\,m$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the seesaw plank.
Decide the reference point about which the torques will be calculated.	The reference point is the position of the boy at Y.
Decide which force causes the clockwise torque, and which force causes the anticlockwise torque around the chosen reference point.	The force of the girl on the plank provides the clockwise torque. The force of the pivot on the plank provides the anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$
Expand the equation to include each of the torques acting on the seesaw. Note that the boy's torque is not included here as, being the reference point, his torque is zero.	$F_g r_{\perp g} = F_P r_{\perp P}$
Substitute the data into the equation and solve.	$F_g r_{\perp g} = F_P r_{\perp P}$ $294 \times 2.50 = 490 \times 1.50$ $735 = 735$
Identify the magnitude of the clockwise torque compared to the magnitude of the anticlockwise torque.	Around reference point Y (the position of the boy), the clockwise torque due to the girl's position on the plank is equal to the anticlockwise torque due to the pivot's action on the plank. So the plank is in rotational equilibrium.

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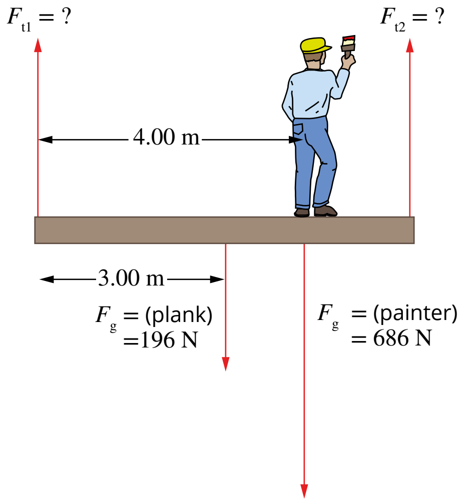
Worked example: Try yourself 3.3.3

CALCULATING STATIC EQUILIBRIUM WITH TWO UNKNOWNS

For the painter on the plank scenario in Worked example 3.3.3, determine the tension on the right-hand rope (F_{t2}).	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	 $m_{pl} = 20.0\,\text{kg}$ $m_{pa} = 70.0\,\text{kg}$ $r_{\perp F_{t2}-F_{t1}} = 6.00\,\text{m}$ $r_{\perp c-F_{t1}} = 3.00\,\text{m}$ $r_{\perp pa-F_{t1}} = 4.00\,\text{m}$ $g = 9.80\,\text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the plank.
Decide the reference point about which the torques will be calculated. Always choose a point at which one of the unknown force acts in order to form an equation with just one unknown. This avoids the need to solve simultaneous equations.	The reference point is the point at which the rope providing the tension force F_{t1} is attached.
Decide which force causes the clockwise torques and which forces cause the anticlockwise torques around the chosen reference point.	The force of the painter on the plank provides a clockwise torque. The force of gravity on the plank provides another clockwise torque. The tension force of the right-hand rope on the plank provides an anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the plank. The torque of the right-hand rope is not included as it acts through the reference point.	$F_{t2}r_{\perp F_{t2}-F_{t1}} = F_{pl}r_{\perp c-F_{t1}} + F_{pa}r_{\perp pa-F_{t1}}$
Substitute the data into the equation and solve for the unknown force.	$F_{t2} \times 6.00 = 20 \times 9.80 \times 3.00 + 70 \times 9.80 \times 4.00$ $F_{t2} = \frac{20 \times 9.80 \times 3.00 + 70 \times 9.80 \times 4.00}{6.00}$ $= \frac{588 + 2744}{6.00}$ $F_{t2} = 555\,\text{N}$

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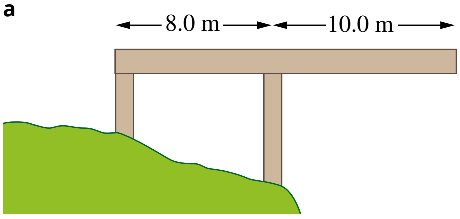
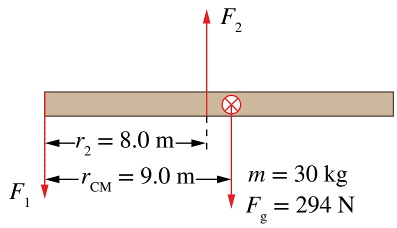
Worked example: Try yourself 3.3.4

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER

a



b



Determine the magnitude and direction of the force that the left-hand support must supply so that the beam is in static equilibrium (F_1).

Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 30.0\text{ kg}$ $r_{\perp F_2-F_1} = 8.00\text{ m}$ $r_{\perp c-F_2} = 1.00\text{ m}$ $g = 9.80\text{ N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object upon which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Decide the reference point about which the torques will be calculated. Choose the point on which the other unknown force acts to eliminate it as an unknown in the equation.	The reference point is the point at which the support providing the force F_2 is attached.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of gravity on the beam provides the anticlockwise torque. The force of the left-hand support on the beam provides the clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam. The torque of the left-hand support is not included, as it acts through the reference point.	$F_b r_{\perp c-F_2} = F_1 r_{\perp F_2-F_1}$
Substitute the data into the equation and solve for the unknown.	$30 \times 9.80 \times 1.00 = F_1 \times 8.00$ $F_1 = \frac{30 \times 9.80 \times 1.00}{8.00}$ $= \frac{294}{8.00}$ $F_1 = 36.8\text{ N}$
State the direction of the force acting on the object in equilibrium.	The force of 36.8 N is downwards on the beam.

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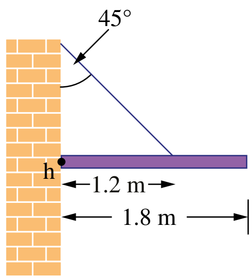
Worked example: Try yourself 3.3.5

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER SUPPORTED BY A TIE

A uniform 5.00 kg beam, 1.80 m long, extends from the side of a building and is supported by a wire tie that is attached to the beam 1.20 m from a hinge (h) at an angle of 45°.



Calculate the tension (F_1) in the wire that is supporting the beam.

Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 5.00$ kg $r_{\perp c\text{–}h} = 0.90$ m $r_{\perp w\text{–}h} = 1.20$ m $g = 9.80$ N kg^{-1}
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Decide the reference point about which the torques will be calculated. Choose the point at which the other unknown force acts to eliminate it from the equation.	The reference point is the hinge (h) at which the beam is connected to the wall.
Decide which force causes the anticlockwise torque, and which force causes the clockwise torques around the chosen reference point.	The force of the wire tie on the beam provides the anticlockwise torque. The force of gravity on the beam provides a clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam.	$F_b r_{\perp c\text{–}h} = F_t r_{\perp W\text{–}h}$
Substitute the data into the equation to solve for the perpendicular distances from the force arm to the line of action of the force.	$r_{\perp w\text{–}h} = r_{w\text{–}h} \sin 45°$ $= 1.2 \times \sin 45°$ $= 0.849$ m
Substitute the data into the equation and solve for the unknown force.	$5.00 \times 9.80 \times 0.900 = F_t \times 1.2 \times \sin 45°$ $F_t = \frac{44.1}{0.849}$ $F_t = 52.0$ N

3.3 Review

1 C. There is no net torque around the reference point; therefore the rate of rotation does not increase when the object is in rotational equilibrium.

2 $F_a r_{\perp a} = F_c r_{\perp c}$

$75.0 \times 9.80 \times r_{\perp a} = 25.0 \times 9.80 \times 2.25$

$r_{\perp a} = \frac{25.0 \times 9.80 \times 2.25}{75.0 \times 9.80}$

$r_{\perp a} = 0.750$ m

The adult must sit 0.750 m from the pivot.

3 B. In order for an object to experience static equilibrium, it must be in both rotational equilibrium and static equilibrium.

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4 $F_a r_{\perp a} = F_{c1} r_{\perp c1} + F_{c2} r_{\perp c2}$
$90.0 \times 9.80 \times r_{\perp a} = 20.0 \times 9.80 \times 1.50 + 20.0 \times 9.80 \times 2.50$
$r_{\perp a} = 0.889\,\text{m}$
The adult must sit 0.889 m from the pivot.

5 $F_t r_{\perp tw\text{–}X} = F_{bs} r_{\perp bs\text{–}X}$
$F_t \times 1.00 = 800 \times 0.85 \cos 60°$
$F_t = \dfrac{800 \times 0.85 \cos 60°}{1.00}$
$F_t = 340\,\text{N}$

6 $F_{left} = 1420\,\text{N}$; $F_{right} = 2990\,\text{N}$
$F_L r_{\perp L\text{–}R} = F_{100} r_{\perp 100\text{–}R} + F_{150} r_{\perp 150\text{–}R} + F_{200} r_{\perp 200\text{–}R}$
$F_L \times 10.0 = 100 \times 9.80 \times 7.00 + 150 \times 9.80 \times 3.00 + 200 \times 9.80 \times 1.50$
$F_L = \dfrac{6860 + 4410 + 2940}{10.0}$
$F_L = 1421\,\text{N} \approx 1420\,\text{N}$
$F_L + F_R = F_{100} + F_{150} + F_{200}$
$F_R = 100 \times 9.80 + 150 \times 9.80 + 200 \times 9.80 - 1421$
$F_R = 2989\,\text{N} \approx 2990\,\text{N}$

7 $F_X r_{\perp c\text{–}X} = F_Y r_{\perp Y\text{–}R}$
$900 \times 9.80 \times 2.0 = F_Y \times 1.8$
$F_Y = \dfrac{900 \times 9.80 \times 2.0}{1.8}$
$F_Y = 9800\,\text{N}$
$= 9.8 \times 10^3\,\text{N}$

8 Downwards. F_X must act downwards on the awning to provide a balancing anticlockwise torque.

9 $F_Y = F_X + F_{awning}$
$F_X = F_Y - F_{awning}$
$= 9800 - 900 \times 9.80$
$F_X = 980\,\text{N}$
The answer can be confirmed by considering torques.

CHAPTER 3 REVIEW

1 D. When a force acts such that the line of action of the force is directed through the pivot point of the object, then no torque results.

2 B. As Tom's velocity is not changing, the translational acceleration must be zero and Tom will be travelling at a constant velocity.

3 C and D. Static equilibrium exists when an object is in translational and rotational equilibrium. Assuming the cyclist is not moving in relation to the bicycle (standing on the pedals and coasting) both the cyclist and the front cog will be in translational and rotational equilibrium. The rate of rotation of the front and rear wheels is decreasing, so a net torque must be acting on them, which implies that they are not in rotational equilibrium.

4 $\tau = r_\perp F$
$= 0.120 \times 30.0$
$\tau = 3.60\,\text{N m}$

5 $\tau = rF \sin\theta$
$= 1.35 \times 74.0 \sin 60°$
$= 86.516$
$\tau = 86.5\,\text{N m}$

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6 Stability refers to the ability of an object to restore its original static equilibrium after being slightly displaced by an outside force.

An object is in unstable equilibrium if the smallest displacement is sufficient to produce a force or torque that continue to make it move away from equilibrium.

An object is in stable equilibrium if the object returns to its original position once the outside force is removed.

An object is in neutral equilibrium if small outside forces don't create any unbalanced forces or torques to move the object further. The object remains in its new equilibrium position.

7 The 75.0 cm spanner. As $\tau = Fr_{\perp}$, the longer the lever arm the less force she will need to exert.

8 Maximum torque will be obtained by applying the 45.0 N force perpendicular to the lever arm at the maximum distance of 75.0 cm from the wheel nut. So:

$\tau = Fr_{\perp}$

$= 45.0 \times 0.750$

$\tau = 33.8\ \text{N m}$

9 **a** The spindle of the tap; about 3 cm

b Front wheel axle of the wheelbarrow; 1 m (handle plus barrow through to the axle)

c The end of the tweezers, usually a few centimetres

d The point of contact between screwdriver and the rim of the tin, 15–30 cm being the length of the screwdriver

10 $\tau = Fr_{\perp}$

$= 2.50 \times 10^3 \times 9.80 \times 20.0$

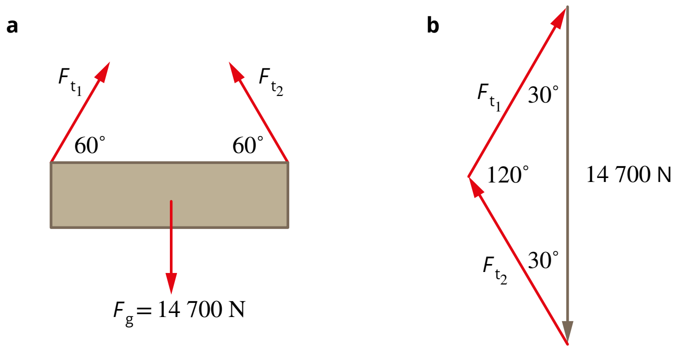
$\tau = 4.9 \times 10^5\ \text{N m}$

11 The crane must have a counterweight providing a torque in the opposite direction.

12 A, B, D. C and E are both experiencing an acceleration due unbalanced forces.

13 There are three forces acting on the beam: the tensile forces acting in each cable F_{t1} and F_{t2}, as well as the weight F_g of the beam. The tensile forces act at each end of the beam and the weight force acts at the centre of mass.

Diagram (a) shows the forces acting on the beam. Diagram (b) shows these forces added as a vector diagram (head to tail) illustrating that they are in static equilibrium.



14 The tension in each cable is the same since points of attachment and angles are mirror images of each other. The beam is moving with a constant velocity, so the forces acting on it are in equilibrium, i.e. $\Sigma F = 0$. Adding the three forces together as vectors (from Question 13) gives an isosceles triangle. The sine rule can be used to solve this (or the triangle can be broken up into right-angled triangles):

$$\frac{F_{t1}}{\sin 30°} = \frac{1500 \times 9.80}{\sin 120°}$$

$$F_{t1} = \frac{14700 \times \sin 30°}{\sin 120°}$$

$F_{t1} = 8487\ \text{N}$

$F_{t2} = F_{t1} = 8.49 \times 10^3\ \text{N}$

8.49 kN in each cable

15 $\Sigma F_{up} = \Sigma F_{down}$

$F_X + F_Y = F_{c1} + F_{c2} + F_{c3} + F_{beam}$

$2.0 \times 10^4 + F_Y = 1000 \times 9.80 + 1500 \times 9.80 + 2000 \times 9.80 + 500 \times 9.80$

$F_Y = 48560 - 20000$

$= 28560$

$F_Y = 2.9 \times 10^4\ \text{N upwards}$

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16 $\Sigma F_{up} = \Sigma F_{down}$

$F_T + F_F = 4F_L$

$40.0 \times 9.80 + 4.50 \times 9.80 = 4F_L$

$436.1 = 4F_L$

$F_L = \frac{436.1}{4}$

$F_L = 109\,\text{N}$ upwards

17 $\Sigma F_{up} = \Sigma F_{down}$

$F_T = F_C$

$F_T = 86.5 \times 9.80$

$F_T = 848\,\text{N}$

18 $F_L = 141\,\text{N}$, $F_R = 123\,\text{N}$

To be in translational equilibrium, the sum of the forces on the *y*-axis must equal zero. Identify all forces acting vertically including the left- and right-hand side cables and the mass of the sign.

$\Sigma F_{up} = \Sigma F_{down}$

$F_L \sin 35° + F_R \cos 70° = 12.5 \times 9.80$

$0.5736 F_L + 0.3420 F_R = 122.5$

Similarly, the sum of forces horizontally must equal zero. Identify those from the left- and right-hand sides.

$F_{left} = F_{right}$

$F_L \cos 35° = F_R \sin 70°$

$0.8191 F_L = 0.9397 F_R$

$F_R = \frac{0.8191 F_L}{0.9397}$

$F_R = 0.8717 F_L$

Substitute this equation for F_R into the equation for the vertical forces, giving:

$0.5736 F_L + 0.3420 \times 0.8717 F_L = 122.5$

$0.5736 F_L + 0.2981 F_L = 122.5$

$0.8717 F_L = 122.5$

$F_L = \frac{122.5}{0.8717}$

$F_L = 140.5\,\text{N}$

$F_L = 141\,\text{N}$

Using the equation for F_R in terms of F_L from the horizontal forces:

$F_R = 0.8717 F_L$

$= 0.8717 \times 140.5$

$F_R = 122.5\,\text{N}$

$F_R = 123\,\text{N}$

19 Sum of the vertical forces must be zero for static equilibrium:

$F_{up} = F_{down}$

$F_T = F_S + F_E + F_R$

$= 0.145 \times 9.80 + 0.0225 \times 9.80 + 0.010 \times 9.80$

$F_T = 1.7395\,\text{N}$

With the bar in static equilibrium and using the model of the Sun as the reference point to eliminate it from the equation:

$\Sigma \tau_{clockwise} = \Sigma \tau_{anticlockwise}$

$F_R r_{\perp R} + F_E r_{\perp E} = F_T r_{\perp T}$

$0.0100 \times 9.80 \times 0.500 + 0.0225 \times 9.80 \times 1.00 = 1.7395 \times r_{\perp T}$

$0.2695 = 1.7395 \times r_{\perp T}$

$r_{\perp T} = \frac{0.2695}{1.7395}$

$r_{\perp T} = 0.155\,\text{m}$

$r_{\perp T} = 15.5\,\text{cm}$

It needs to be tied 15.5 cm from the model of the Sun.

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Chapter 4 Electric fields

Section 4.1 Electric fields

Worked example: Try yourself 4.1.1

USING $F = qE$

Calculate the magnitude of the uniform electric field that creates a force of 9.00×10^{-23} N on a proton. ($q_p = +1.602 \times 10^{-19}$ C)	
Thinking	**Working**
Rearrange the relevant equation to make electric field strength the subject.	$F = qE$ $E = \frac{F}{q}$
Substitute the values for F and q into the rearranged equation and calculate the answer.	$E = \frac{9.00 \times 10^{-23}}{1.602 \times 10^{-19}}$ $= 5.62 \times 10^{-4}$ N C^{-1}

4.1 Review

1 C. In an electric field, a force is exerted between two charged objects.

2 B. The electric field direction is defined as being the direction that a positively charged test charge moves when placed in the electric field.

3 **a** True. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface.

b False. Field lines can never cross. If they did it would indicate that the field is in two directions at that point, which can never happen.

c False. Electric fields go from positively charged objects to negatively charged objects.

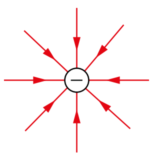
d True. Around small charged spheres called point charges you should draw at least eight field lines: top, bottom, left, right and in between each of these.

e True. Around point charges the field lines radiate like spokes on a wheel.

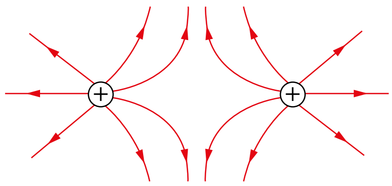
f False. Between two point charges, the direction of the field at any point is the resultant field vector determined by adding the field vectors due to each of the two point charges.

g False. The field between two oppositely charged parallel plates is evenly spaced and is drawn straight from the positive plate to the negative plate.

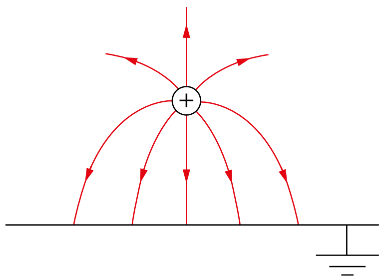
4 **a**



b



c



5 $F = qE$

$= 5.00 \times 10^{-3} \times 2.5$

$= 0.005 \times 2.5$

$= 0.0125$

$= 1.25 \times 10^{-2}$ N

6 $F = qE$

$q = \frac{F}{E}$

$= \frac{0.025}{18}$

$= 0.001\,39$ C

$= 1.39 \times 10^{-3}$ C

$= 1.39$ mC

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7 $F = qE$

$= 1.602 \times 10^{-19} \times 3.25$

$= 5.207 \times 10^{-19}\,\text{N}$

and

$F = ma$

$a = \frac{F}{m}$

$= \frac{5.207 \times 10^{-19}}{9.11 \times 10^{-31}}$

$= 5.72 \times 10^{11}\,\text{m s}^{-2}$

Section 4.2 Coulomb's law

Worked example: Try yourself 4.2.1

USING COULOMB'S LAW TO CALCULATE FORCE

Two small spheres A and B act as point charges separated by 75.0 mm in air. Calculate the force on each point charge if A has a charge of 475 nC and B has a charge of 833 pC.
(Use $\varepsilon_0 = 8.8542 \times 10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$.)

Thinking	Working
Convert all values to SI units.	$q_A = 475 \times 10^{-9} = 4.75 \times 10^{-7}\,\text{C}$ $q_B = 833 \times 10^{-12} = 8.33 \times 10^{-10}\,\text{C}$ $r = 7.50 \times 10^{-2}\,\text{m}$
State Coulomb's law.	$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
Substitute the values q_A, q_B, r and ε_0 into the equation and calculate the answer.	$F = \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{4.75 \times 10^{-7} \times 8.33 \times 10^{-10}}{(7.50 \times 10^{-2})^2}$ $= 6.32 \times 10^{-4}\,\text{N}$
Assign a direction based on a negative force being attraction and a positive force being repulsion.	$F = 6.32 \times 10^{-4}\,\text{N}$ repulsion

Worked example: Try yourself 4.2.2

USING COULOMB'S LAW TO CALCULATE CHARGE

Two small point charges are charged by transferring a number of electrons from q_1 to q_2, and are separated by 12.7 mm in air. The charges are equal and opposite. Calculate the charge on q_1 and q_2 if there is an attractive force of 22.5 μN between them.
(Use $k = 9.0 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$.)

Thinking	Working
Convert all values to SI units.	$F = 22.5 \times 10^{-6} = 2.25 \times 10^{-5}\,\text{N}$ $r = 12.7 \times 10^{-3} = 1.27 \times 10^{-2}\,\text{m}$
State Coulomb's law.	$F = k\frac{q_1q_2}{r^2}$
Substitute the values for F, r and k into the equation and calculate the answer. (Remember to indicate which charge is positive and which is negative in your final answer.)	$q_1q_2 = \frac{Fr^2}{k}$ $= \frac{2.25 \times 10^{-5} \times (1.27 \times 10^{-2})^2}{9.0 \times 10^9}$ $= 4.03 \times 10^{-19}$ Since the magnitudes of q_1 and q_2 are the same: $q_1^2 = 4.03 \times 10^{-19}$ $q_1 = \sqrt{4.03 \times 10^{-19}}$ $q_1 = +6.35 \times 10^{-10}\,\text{C}$ $q_2 = -6.35 \times 10^{-10}\,\text{C}$

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Worked example: Try yourself 4.2.3

ELECTRIC FIELD OF A SINGLE POINT CHARGE

Calculate the magnitude and direction of the electric field at point P at a distance of 15 cm to the right of a positive point charge, q, of 2.0×10^{-6} C.	
Thinking	**Working**
Determine the known and unknown quantities and convert to SI units as required.	$E = ?$ $q = 2.0 \times 10^{-6}$ C $r = 15$ cm $= 0.15$ m
Substitute the known values to find the magnitude of E using $E = k\frac{q}{r^2}$.	$E = k\frac{q}{r^2}$ $= 9.0 \times 10^9 \times \frac{2.0 \times 10^{-6}}{0.15^2}$ $= 8.0 \times 10^5$ N C^{-1}
The direction of the field is defined as that acting on a positive test charge. Point P is to the right of the charge.	Since the charge is positive, the direction will be away from the charge q or to the right.

4.2 Review

1 When a positive charge is multiplied by a negative charge the force is negative, and a positive charge attracts a negative charge. When a negative charge is multiplied by a negative charge the force is positive, and a negative charge repels a negative charge.

Force	q_1 charge	q_2 charge	Action
a positive	positive	positive	repulsion
b negative	negative	positive	attraction
c positive	negative	negative	repulsion
d negative	positive	negative	attraction

2 D. 24.0×10^3 N C^{-1}. Since the distance has been halved, by the inverse square law the field will be four times the original.

3 $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$= \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{+1.602 \times 10^{-19} \times -1.602 \times 10^{-19}}{(53 \times 10^{-12})^2}$

$= -8.22 \times 10^{-8}$ N

4 Recall that $E = k\frac{q}{r^2}$

$E = 9 \times 10^9 \times \frac{3.0 \times 10^{-6}}{(0.05)^2}$

$= 1.1 \times 10^7$ N C^{-1}

5 $F = k\frac{q_1q_2}{r^2}$

$= 9 \times 10^9 \times \frac{1.00 \times 1.00}{1000^2}$

$= 9000$ N

6 $F = mg = 0.01 \times 9.8 = 0.098$ N

$= k\frac{q_1q_2}{r^2}$

$0.098 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{r^2}$

$r^2 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{0.098}$

$= 2.059$

$r = \sqrt{2.059}$

$= 1.435$ m

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7 The force is directly proportional to the product of the charges. The force is inversely proportional to the square of the distance between the point charges.

a If one of the charges is doubled to $+2q$, the force will **double** and **repel**.

b If both charges are doubled to $+2q$, the force will **quadruple** and **repel**.

c If one of the charges is changed to $-2q$, the force will **double** and **attract**.

d If the distance between the charges is halved to $0.5r$, the force will **quadruple** and **repel**.

8 There are two protons within the helium nucleus. Recall that a proton has a charge of $q_p = +1.602 \times 10^{-19}$ C. Use Coulomb's law to calculate the force on the protons.

$$F = k\frac{q_1q_2}{r^2}$$

$$= 9 \times 10^9 \times \frac{1.602 \times 10^{-19} \times 1.602 \times 10^{-19}}{(2.5 \times 10^{-15})^2}$$

$$= 36.96\,\text{N}$$

$$= 37\,\text{N}$$

9 Determine the charge on either point using Coulomb's law:

$$F = k\frac{q_1q_2}{r^2}$$

$$1 = 9 \times 10^9 \times \frac{q^2}{(0.30)^2}$$

$$q = \sqrt{\frac{1 \times 0.30^2}{9 \times 10^9}} = 3.16 \times 10^{-6}\,\text{C}$$

Since each electron has charge 1.602×10^{-19} C, the number of electrons $= \frac{3.16 \times 10^{-6}}{1.602 \times 10^{-19}}$

$= 1.97 \times 10^{13}$ electrons

Section 4.3 Work done in an electric field

Worked example: Try yourself 4.3.1

WORK DONE ON A CHARGE IN A UNIFORM ELECTRIC FIELD

A student sets up a parallel plate arrangement so that one plate is at a potential of 36.0 V and the other earthed plate is positioned 2.00 m away. Calculate the work done to move an electron a distance of 75.0 cm towards the negative plate. ($q_e = -1.602 \times 10^{-19}$ C)

In your answer identify what does the work and what the work is done on.

Thinking	Working
Identify the variables presented in the problem to calculate the electric field strength E.	$V_2 = 36.0$ V $V_1 = 0$ V $d = 2.00$ m
Use the equation $E = \frac{V}{d}$ to determine the electric field strength.	$E = \frac{V}{d}$ $= \frac{36.0 - 0}{2.00}$ $= 18.0\,\text{V m}^{-1}$
Use the equation $W = qEd$ to determine the work done. Note that d here is the distance that the electron moves.	$W = qEd$ $= 1.602 \times 10^{-19} \times 18.0 \times 0.750$ $= 2.16 \times 10^{-18}$ J
Determine if work is done on the charge by the field or if work is done on the field.	Since the negatively charged electron would normally move away from the negative plate, work is done on the field.

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4.3 Review

1 $E = \frac{V}{d}$

$4000 = \frac{V}{0.3}$

$V = 4000 \times 0.3 = 1200\,V$

2 As the oil drop is stationary, the electric force must be equal to the gravitational force. Use the equations $F = mg$ and $F = qE$ to determine the force and the charge respectively. The number of electrons is found by dividing the charge by the charge of one electron.

$F = mg$

$= 1.161 \times 10^{-14} \times 9.8$

$= 1.138 \times 10^{-13}\,N$

$q = \frac{F}{E}$

$= \frac{1.138 \times 10^{-13}}{3.55 \times 10^{4}}$

$= 3.206 \times 10^{-18}\,C$

Number of electrons is found by dividing this value by the charge on one electron:

$= \frac{3.206 \times 10^{-18}}{1.602 \times 10^{-19}}$

= 20 electrons

3 **a** Work is done by the field.

b No work is done.

c Work is done on the field.

d No work is done.

e Work is done on the field.

f Work is done by the field.

4 **a** $W = qEd$

$= 3.204 \times 10^{-19} \times 34 \times 0.01$

$= 1.09 \times 10^{-19}\,J$

b Work is done on the field if the charge is forced to go in a direction it would not naturally go. Alpha particles carry a positive charge, so work is done on the field since a positively charged particle is being moved towards a positive potential.

5 $W = qEd$

$q = \frac{W}{Ed}$

$= \frac{1.24}{(5.05 \times 10^{7} \times 1.10 \times 10^{-2})}$

$q = 2.23 \times 10^{-6}\,C$

CHAPTER 4 REVIEW

1 $F = qE = 3.00 \times 10^{-3} \times 7.50$

$= 0.003 \times 7.50$

$= 0.0225\,N$

2 D

$E = k\frac{q}{r^2}$

$= 9 \times 10^{9} \times \frac{30 \times 10^{-6}}{(0.30)^2}$

$= 3 \times 10^{6}\,N\,C^{-1}$ upwards

Since the electric field is defined as that acting on a positive test charge, the field direction would be upwards or away from the charge, q.

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3 The electrical potential is defined as the work done per unit charge to move a charge from infinity to a point in the electric field. The electrical potential at infinity is defined as zero. When you have two points in an electric field (*E*) separated by a distance (*d*) that is parallel to the field, the potential difference *V* is then defined as the change in the electrical potential between these two points.

4 $E = \frac{V}{d}$

$1000 = \frac{V}{0.025}$

$V = 1000 \times 0.025 = 25\,\text{V}$

5 C. For a uniform electric field, the electric field strength is the same at all points between the plates.

6 When a positively charged particle moves across a potential difference from a positive plate towards an earthed plate, work is done by the **field** on the **charged particle**.

7 Use the equation for work done in a uniform electric field, $W = qEd$.

$W = qEd$

$= 2.5 \times 10^{-18} \times 556 \times 3.0 \times 10^{-3}$

$= 4.17 \times 10^{-18}\,\text{J}$

8 $E = \frac{V}{d} = \frac{15 \times 10^{3}}{0.12} = 125\,000\,\text{V m}^{-1}$

$F = qE$

$= 1.6 \times 10^{-19} \times 125\,000 = 2 \times 10^{-14}\,\text{N}$

9 **a** The distance between the charges is doubled to $2r$, so the force will **quarter** and **repel**.

b The distance between the charges is halved to $0.5r$, so the force will **quadruple** and **repel**.

c The distance between the charges is doubled and one of the charges is changed to $-2q$, so the force will **halve** and **attract**.

10 Recall that kinetic energy gained by the ion (E_k) is equal to work done (W). Therefore, the velocity can be calculated using the equation $E_k = \frac{1}{2}mv^2$, when the kinetic energy is known.

E_k can be calculated in two steps by using the work done on a charge in a uniform electric field equation, $W = qEd$, and the equation to determine the electric field, $E = \frac{V}{d}$.

$E = \frac{V}{d} = \frac{1000}{0.020} = 50\,000\,\text{V m}^{-1}$

$W = qEd = 3 \times 1.602 \times 10^{-19} \times 50\,000 \times 0.020 = 4.806 \times 10^{-16}\,\text{J}$

$E_k = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 4.806 \times 10^{-16}}{3.27 \times 10^{-25}}}$

$= 5.42 \times 10^{4}\,\text{m s}^{-1}$

11 $F = k\frac{q_1q_2}{r^2}$

$= 9 \times 10^{9} \times \frac{5.00 \times 10^{-3} \times 4.00 \times 10^{-9}}{(2.00)^2}$

$= 0.045\,\text{N}$

12 $F = mg$

$F = k\frac{q_1q_2}{r^2}$

$mg = k\frac{q_1q_2}{r^2}$

$r^2 = k\frac{q_1q_2}{mg}$

$= 9 \times 10^{9} \times \frac{2.25 \times 10^{-3} \times 3.05 \times 10^{-3}}{3 \times 9.8}$

$= 2100$

$r = \sqrt{2100}$

$r = 45.8\,\text{m}$

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13 Find the weight force of the ball using $F = mg$. Then substitute this value into the equation $F = Eq$ to calculate the charge.

$F = mg = 5.00 \times 10^{-3} \times 9.8 = 4.9 \times 10^{-2}\,\text{N}$

$= qE$

$q = \frac{F}{E} = \frac{4.9 \times 10^{-2}}{300.0}$

$= +1.63 \times 10^{-4}\,\text{C}$

The charge must be positive to provide an upwards force in a uniform electric field pointing vertically upwards.

14 $F = k\frac{q_1 q_2}{r^2}$

$= 9 \times 10^9 \times \frac{-1.6 \times 10^{-19} \times -1.6 \times 10^{-19}}{(5.4 \times 10^{-12})^2}$

$= 7.9 \times 10^{-6}\,\text{N}$

15 a $F = Eq$

$= 5.50 \times 10^5 \times 1.6 \times 10^{-19}$

$= 8.80 \times 10^{-14}\,\text{N}$ upwards

b $V = Ed$

$= 5.50 \times 10^5 \times 4.50 \times 10^{-2}$

$= 24.8\,\text{V}$

c $W = qV$

$= 1.6 \times 10^{-19} \times 24.75$

$= 3.96 \times 10^{-15}\,\text{J}$

16 a $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$= \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 5 \times 10^{-6}}{(2 \times 10^{-3})^2}$

$= 1.80 \times 10^5\,\text{N}$ attraction

b $q = \frac{(-16 + 5) \times 10^{-6}}{2}$

$= -5.5 \times 10^{-6}\,\text{C}$ per sphere

c $F = k\frac{q_1 q_2}{r^2}$

$= 9 \times 10^9 \times \frac{(-5.50 \times 10^{-6})(-5.50 \times 10^{-6})}{(6.75 \times 10^{-3})^2}$

$F = 5.98 \times 10^3\,\text{N}$ repulsion

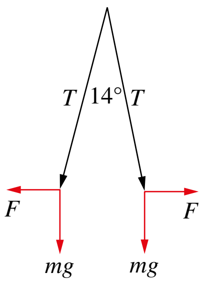
17 a $W = E_k$ gained $= \frac{1}{2}mv^2 = 0.5 \times 9.11 \times 10^{-31} \times (5.30 \times 10^7)^2 = 1.28 \times 10^{-15}\,\text{J}$

b $v = \frac{W}{q}$

$= \frac{1.28 \times 10^{-15}}{1.6 \times 10^{-19}}$

$= 8.00 \times 10^3\,\text{V}$

18



From vector triangle:

$F = T\sin 7°$ and $mg = T\cos 7°$

Therefore $\frac{F}{mg} = \frac{T\sin 7°}{T\cos 7°}$

$F = mg\tan 7°$

$= 5.65 \times 10^{-6} \times 9.8 \times \tan 7°$

$= 6.80 \times 10^{-6}\,\text{N}$

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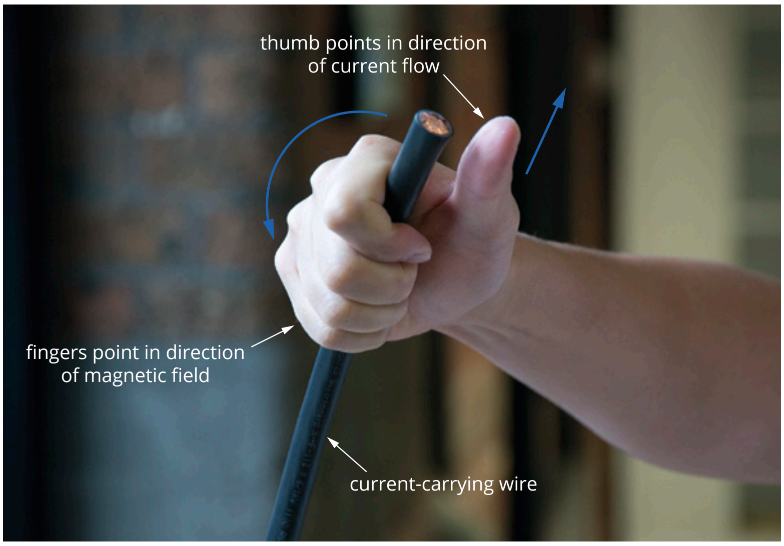
Chapter 5 Magnetic field and force

Section 5.1 The magnetic field

Worked example: Try yourself 5.1.1

DIRECTION OF THE MAGNETIC FIELD

A current-carrying wire runs along the length of a table. The conventional current direction, *I*, is running towards an observer standing at the near end. What is the direction of the magnetic field created by the current as seen by the observer?	
Thinking	**Working**
Recall that the right-hand grip rule indicates the direction of the magnetic field.	Point your thumb to the front in the direction of the current flow. Hold your hand with your fingers aligned as if gripping the wire.  *Source: Alice McBroom*
Describe the direction of the field in relation to the reference object or wire in simple terms, so that the description can be readily understood.	The magnetic field direction is perpendicular to the wire and hence it is circular. As the current travels along the wire, the magnetic field runs anticlockwise around the wire.

Worked example: Try yourself 5.1.2

FLUX DENSITY OF A MAGNETIC FIELD

A current of 4.00 A is running through a long straight wire. Determine the flux density of the magnetic field at a point 3.00 cm from the wire.	
Thinking	**Working**
Identify the quantities supplied in the question and the quantity required. Convert units to SI.	$B = ?$ $I = 4.00\,A$ $r = 3.00\,cm = 0.0300\,m$ $\mu_0 = 4\pi \times 10^{-7}\,T\,m\,A^{-1}$
Substitute quantities into the equation for the flux density of a magnetic field around a wire.	$B = \frac{\mu_0 I}{2\pi r}$ $= \frac{4\pi \times 10^{-7} \times 4.00}{2\pi \times 0.0300}$
Simplify to find *B*.	$B = \frac{2 \times 10^{-7} \times 4.00}{0.0300}$ $B = 2.67 \times 10^{-5}\,T$

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5.1 Review

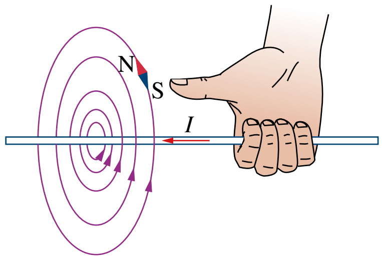
1 B. A north pole is always associated with a south pole. The field around the magnet is known as a dipole field. All magnets are dipolar. This means that every magnet always has a north and a south pole.

2 A. A magnet suspended freely will behave like a compass. The north end of a magnet is attracted to a south pole and the Earth's North Geographic Pole currently coincides with the south pole of the Earth's magnet.

3 C. If you consider the spacing of the magnetic field in the loop as shown by the crosses and dots, it is already non-uniform. Turning the current on and off creates a changing field around the loop, but the loop's magnetic field is still non-uniform.

4 C. The poles are different so the force will be attractive. As distance increases, the force decreases.

5 The direction of the magnetic field created by the current is perpendicular to the wire and runs up in front of the wire then down behind the wire when looking from the front of the wire.



6 The end labelled A is the north pole. Use the right-hand grip rule to find the field in the conductor.

7 **a** A = east, B = south, C = west, D = north

b A = west, B = north, C = east, D = south

8 $B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi \times 0.0300 \times 1.50 \times 10^{-5}}{4\pi \times 10^{-7}}$

$I = 2.25\,\text{A}$

9 $B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi \times 0.250 \times 0.0500 \times 10^{-3}}{4\pi \times 10^{-7}}$

$I = 62.5\,\text{A}$

10 $B = \frac{\mu_0 I}{2\pi r}$

$r = \frac{\mu_0 I}{2\pi B} = \frac{4\pi \times 10^{-7}(95.7 \times 10^{-3})}{2\pi(1.00 \times 10^{-6})}$

$= 0.0191\,\text{m}$

$r = 1.91\,\text{cm}$

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Section 5.2 Forces on charged objects

Worked example: Try yourself 5.2.1

MAGNITUDE OF FORCE ON A POSITIVELY CHARGED PARTICLE

A single, positively charged particle with a charge of $+1.6 \times 10^{-19}$ C travels at a velocity of $50\,m\,s^{-1}$ perpendicular to a magnetic field, B, of flux density 6.0×10^{-5} T.

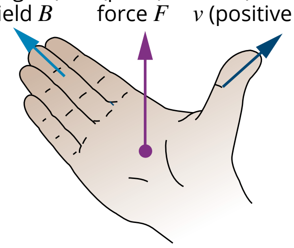
What is the magnitude of the force the particle will experience from the magnetic field?

Thinking	Working
Check the direction of the velocity and determine whether a force will apply. Forces only apply on the component of the velocity perpendicular to the magnetic field.	Particle is moving perpendicular to the field. A force will apply and so $F = qvB$.
Establish which quantities are known and which quantities are required.	$F = ?$ $q = +1.6 \times 10^{-19}$ C $v = 50\,m\,s^{-1}$ $B = 6.0 \times 10^{-5}$ T
Substitute values into the force equation.	$F = qvB$ $= 1.6 \times 10^{-19} \times 50 \times 6.0 \times 10^{-5}$
Express the final answer in appropriate form with appropriate significant figures. Note that only magnitude has been requested, so do not include direction.	$F = 4.8 \times 10^{-22}$ N

Worked example: Try yourself 5.2.2

DIRECTION OF FORCE ON A NEGATIVELY CHARGED PARTICLE

A single, negatively charged particle with a charge of -1.6×10^{-19} C is travelling horizontally from left to right across a computer screen and perpendicular to a magnetic field, B, that runs vertically down the screen. In what direction will the force experienced by the charge act?

Thinking	Working
The right-hand palm rule is used to determine the direction of the force on a positive charge.  	Align your hand so that your fingers are pointing downwards in the direction of the magnetic field. If the negatively charged particle is travelling from left to right, a positively charged particle would be moving in the opposite direction, i.e. from right to left. Align your thumb so it is pointing left, in the direction in which a positive charge would travel. Your palm is facing outwards, which is the direction of the force applied by the magnetic field on a negative charge.

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5.2 Review

1 D. Orientating the right hand with the fingers pointing right and the thumb pointing inwards in the direction of the motion of the charge, the palm is pointing vertically down.

2 **a** South (S). The palm of the hand will be pointing downwards, indicating that the force will be south, based on the compass directions provided.

b The path followed is therefore path C.

c Since v is constant and energy is a scalar quantity, the kinetic energy remains constant.

d Path A. The palm of the hand will be pointing upwards, indicating that the force will be north, based on the compass directions provided. The particle will curve upwards, as the force changes direction with the changing direction of the negative particle.

e Particles with no charge, e.g. neutrons, could follow path B.

3 D.

$F = qvB$

$= 1.6 \times 10^{-19} \times 0.5 \times 2 \times 10^{-5}$

$= 1.6 \times 10^{-24}$ N

4 0 N. There will be no force acting on the particle because the particle is moving parallel to the magnetic field, i.e. there is no component of the motion perpendicular to the field.

5 $F = qvB$

$= 1.6 \times 10^{-19} \times 2 \times 1.5 \times 10^{-5}$

$= 4.8 \times 10^{-24}$ N south

6 The force will double when the velocity doubles. The magnitude of the force becomes $2F$. The direction of the force is north.

7 Charged particles experience a force from the magnetic field that is perpendicular to the particle's velocity, constantly accelerating the charged particle towards the centre. Thus the magnetic force provides the centripetal force.

8 $F = qvB$

$v = \frac{F}{qB} = \frac{9.60 \times 10^{-23}}{(3.20 \times 10^{-19})(3.00 \times 10^{-5})}$

$= 10.0 \text{ m s}^{-1}$

9 $F = qvB$

$B = \frac{F}{qv} = \frac{3.20 \times 10^{-16}}{(1.60 \times 10^{-19})(4.00 \times 10^{4})}$

$= 0.0500$ T

10 The field would need to be halved so that the force $F = qvB$ remains constant, i.e. $F = qvB = q(2v)\left(\frac{B}{2}\right)$.

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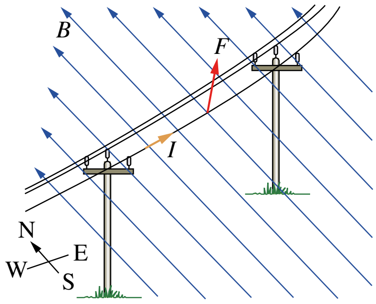
Section 5.3 The force on a conductor

Worked example: Try yourself 5.3.1

MAGNITUDE OF THE FORCE ON A CURRENT-CARRYING WIRE

Determine the magnitude of the force per metre due to the Earth's magnetic field that acts on a suspended power line running east–west near the equator at the moment it carries a current of 50 A from west to east. Assume that the flux density of the Earth's magnetic field at this point is 5.0×10^{-5} T.



Thinking	Working
Check the direction of the conductor and determine whether a force will apply. Forces only apply to the component of the wire perpendicular to the magnetic field.	As the current is running east–west and the Earth's magnetic field runs south–north, the current and the field are at right angles and a force will exist.
Establish which quantities are known and which are required. Since the force per metre is being considered, use a length of 1 m.	$F = ?$ $n = 1$ $I = 50$ A $\ell = 1.0$ m $B = 5.0 \times 10^{-5}$ T
Substitute values into the force equation and simplify.	$F = nI\ell B$ $= 1 \times 50 \times 1.0 \times 5.0 \times 10^{-5}$ N $= 2.5 \times 10^{-3}$ N
Express the final answer in an appropriate form with a suitable number of significant figures. Note that only magnitude has been requested, so do not include direction.	$F = 2.5 \times 10^{-3}$ N per metre of power line

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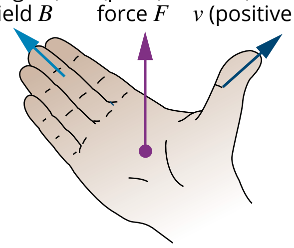
Worked example: Try yourself 5.3.2

DIRECTION OF THE FORCE ON A CURRENT-CARRYING WIRE

A current balance is used to measure the force from a magnetic field on a wire of length 5.0 cm running perpendicular to the magnetic field. The conventional current direction in the wire is from left to right. The magnetic field can be considered to be running out of the page. What is the direction of the force on the wire?	
Thinking	**Working**
The right-hand palm rule is used to determine the direction of the force. 	Align your hand so that your fingers are pointing in the direction of the magnetic field. Align your thumb so it is pointing to the right in the direction of the current. Your palm is facing downwards.
State the direction in terms of the other directions included in the question. Make the answer as clear as possible to avoid any misunderstanding.	The force on the charge is acting vertically downwards.

Worked example: Try yourself 5.3.3

FORCE AND DIRECTION ON A CURRENT-CARRYING WIRE

Santa's house sits at a point that can be considered the Earth's North Magnetic Pole (which behaves like the south pole of a magnet).

Assume the flux density of the Earth's magnetic field at this point is 5.0×10^{-5} T.

a Calculate the magnitude and direction of the magnetic force on a 2.0 m length of wire carrying a conventional current of 10.0 A vertically up the outside wall of Santa's house.	
Thinking	**Working**
Forces only apply to the components of the wire running perpendicular to the magnetic field. The direction of the magnetic field at Earth's North Magnetic Pole will be almost vertically downwards.	The section of the wire running up the wall of the building will be parallel to the magnetic field, *B*. Hence, no force will apply.
State your answer. A numeric value is required. No direction is required with a zero answer.	$F = 0$ N
b Calculate the magnitude and direction of the magnetic force on a 2.0 m length of wire carrying a conventional current of 10.0 A running horizontally right to left across the outside of Santa's house.	
Thinking	**Working**
Forces only apply to the components of the wire running perpendicular to the magnetic field. The direction of the magnetic field at Earth's North Magnetic Pole will be almost vertically downwards.	The section of the wire horizontally across the building will be perpendicular to the magnetic field, *B*. Hence, a force will apply.
Identify the known quantities and the quantity required.	$F = ?$ $n = 1$ $I = 10.0$ A $\ell = 2.0$ m $B = 5.0 \times 10^{-5}$ T
Substitute into the appropriate equation and simplify.	$F = nI\ell B$ $= 1 \times 10.0 \times 2.0 \times 5.0 \times 10^{-5}$ $= 1.0 \times 10^{-3}$ N

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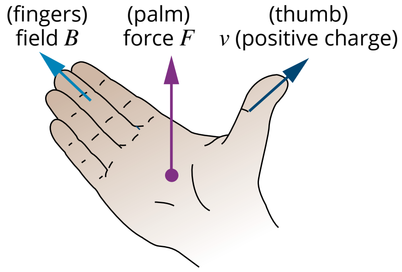
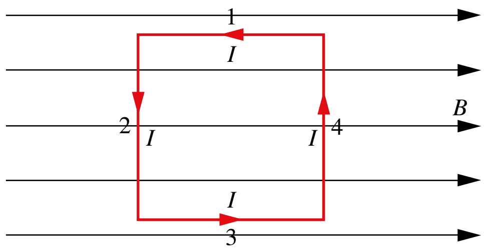
The direction of the magnetic force is also required to fully specify the vector quantity. Determine the direction of the magnetic force using the right-hand palm rule. 	Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. vertically down. Align your thumb so it is pointing left in the direction of the current. Your palm should be facing outwards (out from the house). That is the direction of the force applied by the magnetic field on the wire.
State the magnetic force in an appropriate form with a suitable number of significant figures and with the direction to fully specify the vector quantity.	$F = 1.0 \times 10^{-3}$ N out from Santa's house

Worked example: Try yourself 5.3.4

TORQUE ON A COIL

A single wire coil with a side length 4.0 cm, and free to rotate, sits within a magnetic field, B, of flux density 1.0×10^{-4} T. A current of 1.0 A is flowing through the coil. What is the torque on the coil? Give your answer to two significant figures.



Thinking	Working
Confirm that the coil will experience a force based on the magnetic field and current directions supplied.	Using the right-hand palm rule confirms that a downwards force applies on side 4. An upwards force applies to side 2. The coil will turn clockwise. Sides 1 and 3 lie parallel to the magnetic field and no force will apply.
Calculate the magnetic force on one side.	$F = I\ell B$ $= 1.00 \times 0.040 \times 1.0 \times 10^{-4}$ $F = 0.040 \times 10^{-4}$ N
Determine the distance, r, from the point of rotation that the magnetic force is applied.	Length of side = 4.0 cm Length of rotation = $\frac{1}{2} \times$ side length $r = 2.0$ cm $= 0.020$ m
Calculate the torque applied by the magnetic force on one side of the coil.	$\tau = r_{\perp}F$ $= 0.020 \times 0.040 \times 10^{-4}$ $= 0.00080 \times 10^{-4}$ $\tau = 8.0 \times 10^{-8}$ N m
Since sides 2 and 4, both experience a magnetic force and hence a torque, the torque on one side should be multiplied by 2 to find the total torque. State also the direction of rotation.	Total = $2 \times 8.0 \times 10^{-8}$ N m $\tau = 1.6 \times 10^{-7}$ N m The direction is clockwise.

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5.3 Review

1 The section of the wire PQ runs parallel to the magnetic field, B. Hence, no force will apply. You can confirm this using the right-hand palm rule. Try to align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. to the right, and your thumb is also pointing to the right. It's impossible to do so while still keeping both thumb and fingers outstretched.

2 $F = nI\ell B$

$= 1 \times 80 \times 100 \times 5.0 \times 10^{-5}$

$= 0.4\,\text{N}$

Direction: thumb points right (west to east), fingers point into the page (north), palm will face up

Therefore the force is 0.4 N upwards.

3 **a** $F = nI\ell B$

$= 1 \times 50 \times 80 \times 4.5 \times 10^{-5}$

$= 0.18\,\text{N}$ downwards

b Same as (a). The change in height has no effect on the perpendicular components of the magnetic field (south–north) and the wire's direction.

4 $F = nI\ell B$

$= 1 \times 1.0 \times 0.50 \times 0.20$

$= 0.1\,\text{N}$

5 Current flows into brush P and around the coil from V to X to Y to W. So force on side VX is down, force on side YW is up, so rotation is anticlockwise.

6 D. As $F = nI\ell B$, the coil will experience more force, and rotate faster, if the current and magnetic flux density are increased. Therefore, A and B are correct. Whether C is correct will depend on how the area is increased. But if the length in the field is increased, you would expect it to turn faster. If widened, it will experience more torque but that may not make it turn faster.

7 **a** The direction of the magnetic force is down the page.

b The direction of the magnetic force is up the page.

8 The rotation is anticlockwise.

9 **a** The direction of the magnetic force is down the page.

b The direction of the magnetic force is up the page.

c Zero torque acts as the forces are trying to pull the coil apart rather than turn it. The force is parallel to the coil, rather than perpendicular to it.

10 C. Reversing the direction of the current in the loop will ensure that the loop keeps travelling in the same direction. Use the right-hand palm rule to verify this.

CHAPTER 5 **REVIEW**

1 Based on the directions provided, the direction of the magnetic field would be east—away from the north pole of the left-hand magnet.

2 Based on the directions provided, the direction of the magnetic field would be west—away from the north pole of the right-hand magnet.

3 A magnetic field is a vector. If a point is equidistant from two magnets and the directions of the two fields are opposite, then the vector sum would be zero.

4 With the current turned off the loop is producing no field. The steady field in the region would be the only contributing field. It has a value of B into the page.

5 With the current doubled, the loop is producing double the field, $2B$. The steady field in the region would be contributing B. The total is $3B$ into the page.

6 The field from the loop would exactly match that of the field in the region but in the opposite direction. The vector total would be zero.

7 $B = \frac{\mu_0 I}{2\pi r}$

$r = \frac{\mu_0 I}{2\pi B} = \frac{4\pi \times 10^{-7}(3.00)}{2\pi(4.00 \times 10^{-5})}$

$= 0.0150\,\text{m}$

$r = 1.50\,\text{cm}$

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8 $B = \frac{\mu_0 I}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 15}{2\pi(15 \times 10^{-2})}$

$= 2.0 \times 10^{-5}$ T

$B = 20\,\mu$T

9 $B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(30.0 \times 10^{-2})(5.00 \times 10^{-5})}{4\pi \times 10^{-7}}$

$I = 75$ A

10 D. The magnitude of the magnetic force on a conductor aligned so that the current is running parallel to a magnetic field is zero.

A component of the conductor's length must be perpendicular to a magnetic field for a force to be created.

11 **a** The palm indicates the direction of the magnetic force.

b The fingers indicates the direction of the magnetic field.

c The thumb indicates the direction of the conventional current.

12 For the electron beams to behave as shown in (a), v_1 is **equal to** v_2 and the region of the magnetic field, B_y, must be acting **into** the page.

13 $F = nI\ell B$

$0.800 = 1 \times I \times 3.2 \times 0.0900$

$I = \frac{0.800}{0.0900 \times 3.20} = 2.78$ A

14 In each case the force is found from $F = nI\ell B$, as the field is perpendicular to the current.

a $F = 1 \times 1 \times 10^{-3} \times 5 \times 10^{-3} \times 1 \times 10^{-3}$

$= 5.0 \times 10^{-9}$ N into the page (from the right-hand palm rule)

b $F = 1 \times 2 \times 1 \times 10^{-2} \times 0.1 = 2.0 \times 10^{-3}$ N into the page

15 The magnitude of the magnetic force exerted on the electron is:

$F = qvB$

$= 1.6 \times 10^{-19} \times 7.0 \times 10^{6} \times 8.6 \times 10^{-3}$

$= 9.6 \times 10^{-15}$ N

16 Using the right-hand palm rule: fingers point to the right in the direction of the magnetic field, thumb points into the page in the direction of conventional current. The palm points down, indicating the direction of the force is downwards.

17 The east–west line would experience the greater magnetic force because it runs perpendicular to the Earth's magnetic field.

18 $F = nI\ell B$

$= 1 \times 2 \times 0.05 \times 2.0 \times 10^{-3}$

$= 2.0 \times 10^{-4}$ N

Direction is north.

19 A. Recall the equation $\tau = r_{\perp}F$. The maximum torque exists when the force is applied at right angles to the axis of rotation.

20 $F = nI\ell B$

$= 1 \times 2.0 \times 0.05 \times 0.10$

$= 1.0 \times 10^{-2}$ N into the page

21 $F = nI\ell B$

$= 1 \times 2.0 \times 0.05 \times 0.10$

$= 1.0 \times 10^{-2}$ N out of the page

22 The force will be 0 N, as side PQ is parallel to the magnetic field.

23 Considering the direction of the forces acting on sides PS and QR, the coil would rotate in an anticlockwise direction.

24 D. The direction of the current does not affect the magnitude of the torque. Of the options available, this is the only one that doesn't affect either the distance to the axis of rotation or the magnetic force.

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25 $\tau = r_{\perp}F \times 2$ sides

$= 2r_{\perp}F$

$= 2 \times \frac{0.02}{2} \times 1.0 \times 10^{-2}$

$= 2.0 \times 10^{-4}\,\text{N m}$

26 The function of the commutator is to reverse the direction of the current in the coil every half turn, to keep the coil rotating in the same direction.

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Chapter 6 Magnetic field and emf

Section 6.1 Induced emf in a conductor moving in a magnetic field

Worked example: Try yourself 6.1.1

ELECTROMOTIVE FORCE ACROSS AN AIRCRAFT'S WINGS

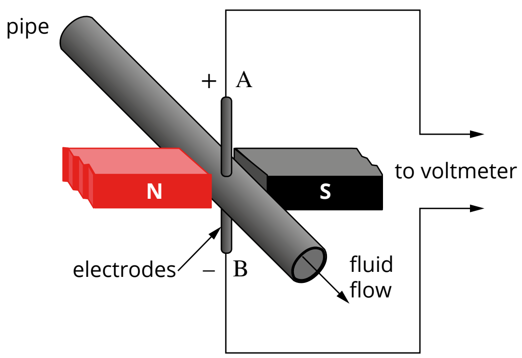
Will a moving aeroplane develop a dangerous emf between its wing tips solely from the Earth's magnetic field?
A fighter jet with a wing span of 25.0 m is flying at a speed of 2000 km h^{-1} at right angles to the Earth's magnetic field of 5.00×10^{-5} T.

Thinking	Working
Identify the quantities required in the correct units.	$\varepsilon = ?$ $\ell = 25.0\ \text{m}$ $B = 5.00 \times 10^{-5}\ \text{T}$ $v = 2000\ \text{km h}^{-1}$ $= 2000 \times \frac{1000}{3600}$ $= 556\ \text{m s}^{-1}$
Substitute into the appropriate formula and simplify.	$\varepsilon = \ell vB$ $= 25.0 \times 556 \times 5.00 \times 10^{-5}$ $= 0.695\ \text{V}$
State your answer as a response to the question.	$\varepsilon = 0.695\ \text{V}$ This is a very small emf and would not be dangerous.

Worked example: Try yourself 6.1.2

FLUID FLOW MEASUREMENT

The rate of fluid flow within a vessel can be measured using the induced emf when the fluid contains charged ions. A small magnet and a sensitive voltmeter calibrated to measure speed are used. This can be applied to measure fluid flow in small pipes. If the diameter of a particular small pipe is 1.00 cm, the strength of the magnetic field applied is 0.100 T and the measured emf is 0.500 mV, what is the speed of the fluid flow?



Thinking	Working
Identify the quantities required and put them into SI units.	$\varepsilon = 0.500\ \text{mV} = 5.00 \times 10^{-4}\ \text{V}$ $\ell = 1.00\ \text{cm} = 1.00 \times 10^{-2}\ \text{m}$ $v = ?$ $B = 0.100\ \text{T}$

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Rearrange the appropriate formula, substitute and simplify.	$\varepsilon = \ell vB$ $v = \frac{\varepsilon}{\ell B}$ $= \frac{5.00 \times 10^{-4}}{1.00 \times 10^{-2} \times 0.100}$ $= 0.500\,\text{m}\,\text{s}^{-1}$
State your answer with the correct units.	The speed of the fluid flow is $0.500\,\text{m}\,\text{s}^{-1}$.

Worked example: Try yourself 6.1.3

EMF ACROSS CONDUCTOR MOVING AT AN ANGLE TO THE MAGNETIC FIELD

A car driving from east to west at $100\,\text{km}\,\text{h}^{-1}$ has a vertical aerial of length 50.0 cm on its roof where the Earth's magnetic field is 5.00×10^{-5} T at an angle of dip of 30.0°. Calculate the induced emf between its ends.	
Thinking	**Working**
Identify the component of magnetic field that will cause an emf to be induced.	Vertical rod will not cut vertical component of magnetic field. The horizontal component of the magnetic field needs to be found.
Calculate the horizontal component of the magnetic field.	$B_h = B\cos\theta$ $B_h = 5.00 \times 10^{-5} \cos 30°$ $B_h = 4.33 \times 10^{-5}\,\text{T}$
Identify the quantities required and put them into SI units.	$\varepsilon = ?$ $\ell = 50.0\,\text{cm} = 0.500\,\text{m}$ $v = 100\,\text{km}\,\text{h}^{-1} = 27.8\,\text{m}\,\text{s}^{-1}$ $B_h = 4.33 \times 10^{-5}\,\text{T}$
Use the appropriate formula, substitute and calculate answer.	$\varepsilon = \ell vB$ $\varepsilon = 0.500 \times 27.8 \times 4.33 \times 10^{-5}$ $\varepsilon = 6.02 \times 10^{-4}$
State your answer with the correct units.	$\varepsilon = 6.02 \times 10^{-4}\,\text{V}$

6.1 Review

1 A. There is no change in magnetic flux in this scenario and so there cannot be an induced emf.

2 $\varepsilon = \ell vB = 0.120 \times 0.150 \times 0.800 = 0.0144\,\text{V}$ or $1.44 \times 10^{-2}\,\text{V}$

3 $\varepsilon = \ell vB$

$$v = \frac{\varepsilon}{\ell B}$$
$$= \frac{100 \times 10^{-3}}{13.2 \times 10^{-2} \times 0.900}$$
$$v = 0.842\,\text{m}\,\text{s}^{-1}$$

4 $\varepsilon = \ell vB$

$$\ell = \frac{\varepsilon}{vB}$$
$$\ell = \frac{80.0 \times 10^{-3}}{1.60 \times 0.500}$$
$$= 0.100\,\text{m}$$

5 As the rod is held vertically and dropped downwards through a vertically upwards field, the rod and magnetic field are oriented parallel to each other. No emf will be produced, therefore the correct answer is 0 V. (As the rod has some width, there would be an emf created across this width, but the question specifically dismisses this by stating it is of 'very small diameter.')

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6 $\varepsilon = \ell vB = 20.0 \times 1000 \times 2.50 \times 10^{-5} = 0.500\,\text{V}$

7 $\varepsilon = B\ell v = 2.50 \times 10^{-5} \times 0.750 \times \frac{60.0}{3.6}$

$= 3.13 \times 10^{-4}\,\text{V}$

8 $\varepsilon = \ell vB = 60.0 \times 260 \times 4.00 \times 10^{-5} = 0.624\,\text{V}$

9 Horizontally $v = 260\cos 10.0 = 256\,\text{m s}^{-1}$

$\varepsilon = \ell v_h B_v$

$= 60.0 \times 256 \times 4.00 \times 10^{-5}$

ε due to horizontal motion = 0.614 V

10 Vertically $v = 260\sin 10.0 = 45.1\,\text{m s}^{-1}$

$\varepsilon = \ell v_v B_h$

$= 60.0 \times 45.1 \times 2.00 \times 10^{-5}$

$= 0.0542\,\text{V}$

Total = ε due to horizontal motion + ε due to vertical motion

= 0.614 V + 0.0542 V

Therefore total $\varepsilon = 0.668\,\text{V}$

Section 6.2 Induced emf from a changing magnetic flux

Worked example: Try yourself 6.2.1

MAGNETIC FLUX

A student places a horizontal square coil of wire of side length 4.00 cm into a uniform vertical magnetic field of 0.0500 T. Calculate the magnetic flux ‘threading’ the coil.

Thinking	Working
Calculate the area of the coil perpendicular to the magnetic field.	side length = 4.00 cm = 0.0400 m area of the square = $(0.0400\,\text{m})^2$ = $0.001\,60\,\text{m}^2$
Calculate the magnetic flux.	$\Phi = B_{\perp}A$ $= 0.0500 \times 0.001\,60$ $= 0.000\,080\,0\,\text{Wb}$
State the answer in an appropriate form.	$\Phi = 8.00 \times 10^{-5}\,\text{Wb}$

Worked example: Try yourself 6.2.2

INDUCED EMF IN A COIL

A student winds a coil of area $50.0\,\text{cm}^2$ with 10 turns and places it horizontally in a vertical uniform magnetic field of 0.100 T.

a Calculate the magnetic flux perpendicular to the coil.

Thinking	Working
Identify the quantities to calculate the magnetic flux through the coil and convert to SI units where required.	$\Phi = BA_{\perp}$ $B = 0.100\,\text{T}$ $A = 50.0\,\text{cm}^2 = 50.0 \times 10^{-4}\,\text{m}^2$
Calculate the magnetic flux and give your answer with appropriate units.	$\Phi = BA_{\perp} = 0.100 \times 50.0 \times 10^{-4}$ $= 5.00 \times 10^{-4}\,\text{Wb}$

b Calculate the magnitude of the average induced emf in the coil when the coil is removed from the magnetic field in a time of 1.00 s.

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Identify the quantities for determining the induced emf. Ignore the negative sign.	$\varepsilon = -N\frac{\Delta\Phi}{t}$ $N = 10$ turns $\Delta\Phi = \Phi_2 - \Phi_1$ $= 0.00 - 5.00 \times 10^{-4}$ $= -5.00 \times 10^{-4}$ Wb $t = 1.00$ s
Calculate the magnitude of the average induced emf.	$\varepsilon = -N\frac{\Delta\Phi}{t}$ $= -10 \times \frac{-5.00 \times 10^{-4}}{1.00}$ $= 5.00 \times 10^{-3}$ V

Worked example: Try yourself 6.2.3

NUMBER OF TURNS IN A COIL

A coil of cross-sectional area 2.00×10^{-3} m^2 experiences a change in the strength of a magnetic field from 0.00 to 0.200 T over 1.00 s. If the magnitude of the average induced emf is measured as 0.400 V, how many turns must be on the coil?	
Thinking	**Working**
Identify the quantities needed to calculate the magnetic flux through the coil when in the presence of the magnetic field and convert to SI units where required.	$\Phi = BA_\perp$ $B = 0.200$ T $A = 2.00 \times 10^{-3}$ m^2
Calculate the magnetic flux when in the presence of the magnetic field.	$\Phi = BA_\perp$ $= 0.200 \times 2.00 \times 10^{-3}$ $= 4.00 \times 10^{-4}$ Wb
Identify the quantities from the question required to complete Faraday's law.	$\varepsilon = -N\frac{\Delta\Phi}{t}$ $N = ?$ $\Delta\Phi = \Phi_2 - \Phi_1$ $= 4.00 \times 10^{-4} - 0$ $= 4.00 \times 10^{-4}$ Wb $\Delta t = 1.00$ s $\varepsilon = 0.400$ V
Rearrange Faraday's law and solve for the number of turns on the coil. Ignore the negative sign.	$\varepsilon = -N\frac{\Delta\Phi}{t}$ $N = \frac{\varepsilon t}{\Delta\Phi}$ $= \frac{0.400 \times 1.00}{4.00 \times 10^{-4}}$ $= 1000$ turns

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6.2 Review

1 0 Wb. As the normal to the plane of the coil is perpendicular to the magnetic field, there is no wire cutting through the flux lines.

2 $\Phi = BA_{\perp} = 1.60 \times 10^{-3} \times \pi \times 0.0500^2 = 1.30 \times 10^{-5}$ Wb

3 $\Phi = BA_{\perp} = 2.00 \times 10^{-3} \times 0.0200 \times 0.0300 = 1.20 \times 10^{-6}$ Wb

4 Zero flux threads the loop when the normal to the plane of the loop is perpendicular to the magnetic field.

5 $\Delta\Phi = 1.20 \times 10^{-6}$ Wb, $t = 40.0$ ms $= 40.0 \times 10^{-3}$ s

$$\varepsilon = -N\frac{\Delta\Phi}{t}$$

$$\varepsilon = -1 \times \frac{1.20 \times 10^{-6}}{0.040}$$

$$= 3.00 \times 10^{-5}\ \text{V}$$

6 $\Phi = 80.0 \times 10^{-3} \times 10.0 \times 10^{-4} = 8.00 \times 10^{-5}$ Wb

$$\varepsilon = -N\frac{\Delta\Phi}{t}$$

$$\varepsilon = -1 \times \frac{8.00 \times 10^{-5}}{0.0200}$$

$$= 4.00 \times 10^{-3}\ \text{V}$$

7 $\varepsilon = -N\frac{\Delta\Phi}{t}$

$$\varepsilon = -500 \times \frac{8.00 \times 10^{-5}}{0.0200}$$

$$= 2.00\ \text{V}$$

8 $\varepsilon = -N\frac{\Delta\Phi}{t} = -N\frac{\Delta(BA)}{t}$

$$\varepsilon = -30\,\frac{(5.00 \times 10^{-3})(250 - 50.0) \times 10^{-4}}{0.500}$$

$$= 6.00 \times 10^{-3}\ \text{V}$$

9 $\varepsilon = -N\frac{\Delta\Phi}{t} = -N\frac{\Delta(BA)}{t}$

$$A = \frac{\varepsilon t}{NB} = \frac{0.0200 \times 0.0500}{1 \times 0.100}$$

$$= 0.0100\ \text{m}^2$$

10 $\varepsilon = -N\frac{\Delta\Phi}{t} = -N\frac{\Delta(BA)}{t}$

$$t = N\frac{\Delta(BA)}{\varepsilon} = 100\frac{0.400(50.0 \times 10^{-4})}{1600 \times 10^{-3}}$$

$$= 0.125\ \text{s}$$

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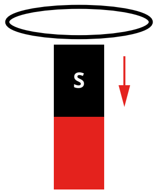
Section 6.3 **Lenz's law**

Worked example: Try yourself 6.3.1

INDUCED CURRENT IN A COIL FROM A PERMANENT MAGNET

The south pole of a magnet is moved downwards away from a horizontal coil held above it. In which direction will the induced current flow in the coil?



Thinking	Working
Consider the direction of the change in magnetic flux.	The magnetic field direction will be downwards towards the south pole. The downwards flux from the magnet will decrease as the magnet is moved away from the coil. So the change in flux is decreasing downwards.
What will oppose the change in flux?	The magnetic field that opposes the change would act downwards.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be clockwise when viewed from above (using the right-hand grip rule).

Worked example: Try yourself 6.3.2

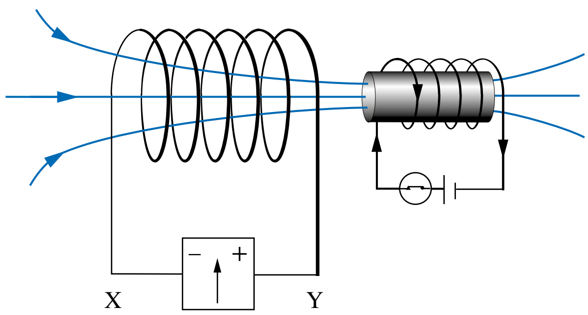
INDUCED CURRENT IN A COIL FROM AN ELECTROMAGNET

What is the direction of the current induced in the solenoid when the electromagnet is:

(i) switched on?

(ii) left on?

(iii) switched off?



Thinking	Working
Consider the direction of the change in magnetic flux for each case.	**(i)** Initially there is no magnetic flux through the solenoid. When the electromagnet is switched on, it creates a magnetic field directed to the right. So the change in flux through the solenoid is increasing to the right. **(ii)** While the current in the electromagnet is steady, the magnetic field is constant and the flux through the solenoid is constant. **(iii)** In this case, initially there is a magnetic field from the electromagnet directed to the right. When the electromagnet is switched off, there is no longer a magnetic field so the change in flux through the solenoid is decreasing to the right.

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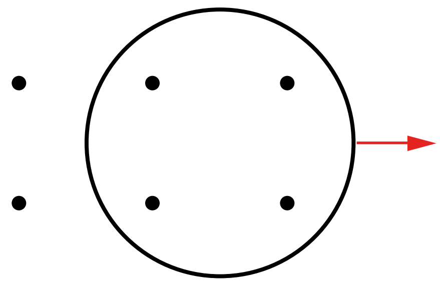
What will oppose the change in flux for each case?	**(i)** The magnetic field that opposes the change in flux through the solenoid is directed to the left. **(ii)** There is no change in flux and so no opposition is needed and there will be no magnetic field created by the solenoid. **(iii)** The magnetic field that opposes the change in flux through the solenoid is directed to the right.
Determine the direction of the induced current required to oppose the change for each case.	**(i)** In order to oppose the change, the current will flow through the solenoid in the direction from Y to X (through the meter from X to Y), using the right-hand grip rule. **(ii)** There will be no induced emf or current in the solenoid. **(iii)** In order to oppose the change, the current will flow through the solenoid in the direction from X to Y (through the meter from Y to X), using the right-hand grip rule.

Worked example: Try yourself 6.3.3

FURTHER PRACTICE WITH LENZ'S LAW

A coil is moved to the right and out of a magnetic field that is directed out of the page. In which direction will the induced current flow in the coil while the magnet is moving?



Thinking	Working
Consider the direction of the change in magnetic flux.	Initially, the magnetic flux passes through the full area of the coil and out of the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing out of the page.
What will oppose the change in flux?	The magnetic field that opposes the change would act out of the page again.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be anticlockwise (using the right-hand grip rule).

6.3 Review

1 C. The magnetic field of the induced current will always oppose the original change.

2 **a** A. When the external magnetic field is switched off this represents a change in flux through the coil that is decreasing out of the page. In order to oppose this change, the induced current will create a magnetic field out of the page.

b A. When the external magnetic field is reversed this represents a change in flux through the coil that is decreasing out of the page, followed by increasing into the page. In order to oppose this change, the induced current will create a magnetic field out of the page.

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3 a Anticlockwise. The magnetic flux is increasing in strength downwards, so the field opposing that change will be upwards. Using the right-hand grip rule, the direction of the induced current will be anticlockwise.
   b Any combination of four factors that address the three ways of creating an induced emf or the relationship between emf and current, such as:
      1 strength of the magnet
      2 speed of the magnet
      3 area/diameter of the ring
      4 orientation of the ring
      5 type of copper making up the ring
      6 resistance of the circuit containing the coil.

4 Lenz's law states that an induced current is created, which produces flux to oppose the change in flux. As the copper wire accelerates through the magnetic field, the induced opposing magnetic field produces a repulsive upwards force that reduces the net force acting on the wire. When the upwards magnetic force is equal to the downwards gravitation force, there is no net force and a constant terminal velocity is reached.

5 The compass will oscillate freely above the glass but damping will occur over the aluminium due to induced eddy currents in the aluminium that create a field to oppose the change of flux, according to Lenz's law.

6 Top end. Right-hand fingers pointing north, palm opposing the motion of the wire, thumb points upwards.

7 Anticlockwise. Right-hand thumb pointing upwards in the same direction as the declining flux, fingers curl anticlockwise around the loop.

8 Up at the front of the solenoid. Right-hand thumb pointing left as a north pole opposed the approaching north pole of the magnet. Fingers curl around the solenoid up at the front and down at the back.

9 Into the page. Right-hand fingers pointing to the right, palm downwards opposing the direction of motion, thumb points into the page.

10 Induction cookers**:** AC current in the coil produces a changing flux, resulting in eddy currents in metal pans (best with ferromagnetic bases); currents will heat up the pan, which transfers heat to its content. Soft-closing kitchen drawers also use eddy currents created by a magnet on the drawer inducing an eddy current in a metal plate. This causes an induced field that opposes the motion of the magnet on the drawer, causing it to slow down as it closes. Melting gold at the Perth Mint is done without the need for fuel and flames by using large alternating voltages that create eddy currents in the gold. The eddy currents heat the gold to beyond its melting point.

Section 6.4 Transforming voltage using changing magnetic field

Worked example: Try yourself 6.4.1

AC GENERATOR

The armature of a 50 Hz AC generator is rotating within a 0.200 T magnetic field. If the coil has 500 turns, what must the area be, in m^2, for the peak output to be 340 V?

Thinking	Working
The question is an application of $\varepsilon = -2\pi NBA_{\perp}f$. Identify the known and unknown quantities in the equation.	$\varepsilon = 340$ V $N = 500$ $B = 0.200$ T $A = ?$ m^2 $f = 50$ Hz
Rearrange the equation in terms of the unknown, N. The negative sign can be left out—it is a reminder of the direction of the induced emf opposing the change that created it.	$\varepsilon = -2\pi NBA_{\perp}f$ $A = \frac{\varepsilon}{2\pi NBf}$ $= \frac{340}{2\pi \times 500 \times 0.2.00 \times 50}$ $= 0.011$ m^2

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Worked example: Try yourself 6.4.2

PEAK AND RMS AC VALUES

A 1000 W kettle is connected to a 240 V AC power outlet. Calculate the peak power use of the kettle.	
Thinking	**Working**
Note that the values given in the question represent rms values. Power is $P = VI$ so both V and I must be known to calculate the power use. The voltage V is given, and the current I can be calculated from the rms power supplied.	$P_{rms} = V_{rms}I_{rms}$ $I_{rms} = \frac{P_{rms}}{V_{rms}}$ $= \frac{1000}{240}$ $= 4.17$ A
Substitute in known quantities and solve for peak power.	$P_{peak} = \sqrt{2}V_{rms} \times \sqrt{2}I_{rms} = 2V_{rms}I_{rms}$ $= 2 \times V_{rms} \times I_{rms}$ $= 2 \times 240 \times 4.17$ $= 2000$ W

Worked example: Try yourself 6.4.3

TRANSFORMER EQUATION—EMF

A transformer is built into a phone charger to reduce the 240 V supply voltage to the required 6.00 V for the charger. If the number of turns in the secondary coil is 100, what is the number of turns required in the primary coil?	
Thinking	**Working**
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$V_s = 6.00$ V $V_p = 240$ V $N_s = 100$ turns $N_p = ?$ $\frac{V_p}{V_s} = \frac{N_p}{N_s}$
Substitute the quantities into the equation, rearrange and solve for N_p.	$\frac{N_p}{100} = \frac{240}{6.00}$ $N_p = \frac{100 \times 240}{6.00}$ $= 4000$ turns

Worked example: Try yourself 6.4.4

TRANSFORMER EQUATION—CURRENT

A phone charger with 4000 turns in the primary coil and 100 turns in its secondary coil draws a current of 0.500 A. What is the current in the primary coil?	
Thinking	**Working**
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$N_s = 100$ turns $N_p = 4000$ turns $I_s = 0.500$ A $I_p = ?$ $\frac{I_p}{I_s} = \frac{N_s}{N_p}$
Substitute the quantities into the equation, rearrange and solve for I_1.	$\frac{I_p}{0.500} = \frac{100}{4000}$ $I_p = \frac{0.500 \times 100}{4000}$ $= 0.0125$ A

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Worked example: Try yourself 6.4.5

TRANSFORMERS—POWER

The power drawn from the secondary coil of a transformer by a phone charger is 3.00 W. What power is drawn from the mains supply if the transformer is an ideal transformer?	
Thinking	**Working**
The energy efficiency of an ideal transformer can be assumed to be 100%. The power in the secondary coil will be the same as that in the primary coil.	The power drawn from the mains supply is the power in the primary coil, which will be the same as the power in the secondary coil: $P = 3.00$ W.

Worked example: Try yourself 6.4.6

TRANSMISSION-LINE POWER LOSS

300 MW is to be transmitted from the Collie power station to Perth along a transmission line with a total resistance of 1.00 Ω. What would be the total transmission power loss if the initial voltage at Collie was now to be 500 kV?	
Thinking	**Working**
Convert the values to SI units.	$P = 300\text{ MW} = 300 \times 10^6\text{ W}$ $V = 500\text{ kV} = 500 \times 10^3\text{ V}$
Determine the current in the line based on the required voltage.	$P = VI$ $\therefore I = \frac{P}{V}$ $I = \frac{300 \times 10^6}{500 \times 10^3}$ $= 600\text{ A}$
Determine the corresponding power loss.	$P_{loss} = I^2R$ $= 600^2 \times 1.00$ $= 3.60 \times 10^5\text{ W}$ or 0.360 MW

Worked example: Try yourself 6.4.7

VOLTAGE DROP ALONG A TRANSMISSION LINE

Power is to be transmitted from the Garden Island wave-power station to Fremantle along a transmission line with a total resistance of 1.00 Ω. The current is 600 A. What initial voltage would be needed at the Garden Island end of the transmission line to achieve a supply voltage of 500 kV?	
Thinking	**Working**
Determine the voltage drop along the transmission line.	$\Delta V = IR$ $= 600 \times 1.00$ $= 600\text{ V}$
Determine the initial supply voltage.	$V_{initial} = V_{supplied} + \Delta V$ $= 500 \times 10^3 + 600$ $= 500\,600\text{ V}$ or 500.6 kV

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6.4 Review

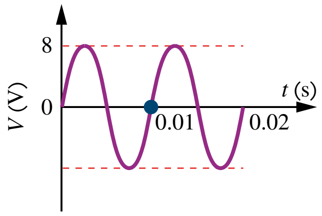
1 B. When the coil begins rotating, the flux is a maximum and decreases initially, having the shape of graph D. The current graph (like the induced emf graph) will be zero initially and will increase, having the pattern shown in graph B.

2 $V_{peak} = 8.00\,V$

$V_{peak-peak} = 2 \times V_p = 2 \times 8.00 = 16.0\,V$

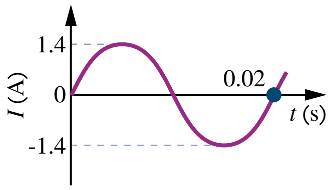
$V_{rms} = \frac{V_p}{\sqrt{2}} = \frac{8.00}{\sqrt{2}} = 5.66\,V$

3



Doubling the magnetic field strength will double the emf, as will doubling the frequency. Halving the radius reduces the area by one-quarter, and so reduces the emf by one-quarter. Thus the emf will remain the same magnitude overall. Doubling the frequency of rotation, however, does reduce the period of the output graph by half.

4



5 D. A change in flux through the secondary coil is required for an emf to be induced in the secondary coil, but a DC input to the primary coil will create a constant flux. Therefore the voltage output is zero.

6 $\frac{N_2}{N_1} = \frac{V_2}{V_1}$

$\frac{N_2}{800} = \frac{12}{240}$

$N_2 = \frac{12 \times 800}{240}$

$= 40$ turns

7 **a** In an ideal transformer there should be no power loss, so $P_1 = P_2$.

b $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

8 $V_p = 8.00\,V$

$I_p = 2.00\,A$

$N_s = 200$ turns

$N_p = 20$

a $\frac{N_p}{N_s} = \frac{8.00}{V_s}$

$V_s = \frac{V_p \times N_s}{N_p} = \frac{8.00 \times 200}{20}$

$= 80.0\,V$

b Assuming it is an ideal transformer, power input from primary is equal to power output to secondary:

$P_p = V_p I_p = 8.00 \times 2.00 = 16.0\,W$

c $P_s = V_s I_s$

$16.0 = 80.0 \times I_s$

$I_s = 0.200\,A$

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9 $P = 5.00 \times 10^3$

$V = 500$ V

$R = 4\ \Omega$

$I = \frac{P}{V} = \frac{5.00 \times 10^3}{500}$

$= 10.0$ A

$P_{loss} = I^2R = 10.0^2 \times 4.00$

$= 400$ W

10 **a** $I = \frac{P}{V} = \frac{500 \times 10^6}{100 \times 10^3} = 5000$ A

b $\Delta V = I \times R$

$= 5000 \times 2 = 10\,000$ V or 10 kV

$V_{supplied} = 100 - 10 = 90$ kV

CHAPTER 6 REVIEW

1 $\Phi = BA_{\perp} = 2.00 \times 10^{-3} \times (4.00 \times 10^{-2})^2$

$\Phi = 3.20 \times 10^{-6}$ Wb

2 The magnetic flux decreases from 3.20×10^{-6} Wb to 0 after one-quarter of a turn. Then it increases again to 3.20×10^{-6} Wb through the opposite side of the loop after half a turn. Then it decreases to 0 again after three-quarters of a turn. After a full turn it is back to 3.20×10^{-6} Wb again.

3 C. The speed of the magnet reduces the time over which the change occurs but there is no change in the strength of the magnetic field or the area of the coil, hence the total flux (area under the curve) is the same.

4 The student must induce an emf of 1.00 V in the wire by somehow changing the magnetic flux through the coil at an appropriate rate. A change in flux can be achieved by changing the strength of the magnetic field or by changing the area of the coil. The magnetic field can be changed by changing the position of the magnet relative to the coil. The area can be changed by changing the shape of the coil or by rotating the coil relative to the magnetic field.

To calculate the required rate of change of flux to produce 1.00 V:

$\varepsilon = -N\frac{\Delta\Phi}{t}$

$\frac{\Delta\Phi}{t} = \frac{\varepsilon}{N} = \frac{1.00}{100}$

$= 0.0100$ Wb s^{-1}

For example, if the shape was changed from 0.0100 m^2 to 0.0200 m^2 then:

$\frac{\Delta\Phi}{t} = \frac{B\Delta A}{t} = \frac{100 \times 10^{-3} \times (0.0200 - 0.0100)}{0.100}$

$= 0.0100$ Wb s^{-1}

5 B. Applying Lenz's law, the back emf opposes the change in magnetic flux that created it, so the induced back emf will be in the opposite direction to the emf creating it. The net emf used by the motor is then less than the supplied voltage.

6 $V_{rms} = \frac{10.0}{\sqrt{2}} = 7.07$ V

$I_{rms} = \frac{6.0}{\sqrt{2}} = 4.24$ A

$P_{rms} = V_{rms} \times I_{rms}$

$= 7.07 \times 4.24 = 29.98$ W

$P_{rms} = 30.0$ W

7 $I_{rms} = \frac{P_{rms}}{V_{rms}} = \frac{600}{240} = 2.50$ A

$I_p = \sqrt{2} \times 2.50 = 3.54$ A

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8 **a** B changes from 8.00×10^{-4} T to 16.0×10^{-4} T, which is a change of 8.00×10^{-4} T.

$\Delta\Phi = \Delta BA = 8.00 \times 10^{-4} \times 40.0 \times 10^{-4}$

$= 3.20 \times 10^{-6}$ Wb

$\varepsilon = -N\frac{\Delta\Phi}{t} = \frac{3.20 \times 10^{-6}}{1.00 \times 10^{-3}} = 3.20 \times 10^{-3}$ V

b Clockwise. Doubling the magnetic field strength increases the flux through the coil out of the page. The induced magnetic field will act into the page to oppose the increasing magnetic flux out of the page. Using the right-hand grip rule, the induced current direction is clockwise around the coil.

9 **a** $A = \pi r^2 = \pi \times (4.00 \times 10^{-2})^2 = 5.02 \times 10^{-3}$ m^2

$\Phi = BA = 20.0 \times 10^{-3} \times 5.02 \times 10^{-3}$

$\Phi = 1.01 \times 10^{-4}$ Wb

$\varepsilon = -N\frac{\Delta\Phi}{t} = 40 \times \frac{0 - 1.01 \times 10^{-4}}{0.100} = 0.0402$ V

b From Y to X. As the coil is removed, the magnetic flux through the coil changes from being directed downwards to no magnetic flux. To oppose this change the coil must create a magnetic field that is directed downwards again. Using the right-hand grip rule, this means the current must flow clockwise around the coil when viewed from above, and from Y to X through the ammeter.

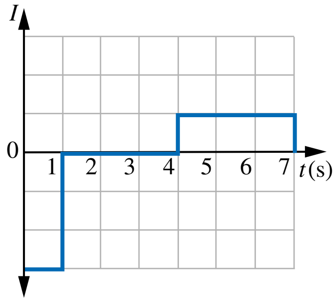
10 **a** $\varepsilon = \ell vB = 0.200 \times 2.00 \times 10.0 \times 10^{-3}$

$\varepsilon = 4.00 \times 10^{-3}$ V $= 4.00$ mV

b From X to Y. As the rod moves to the right, the area of the loop decreases so the magnetic flux through the loop, which is directed out of the page, decreases. In order to oppose this change the loop will create a magnetic field directed out of the page again. Using the right-hand grip rule, the current will flow through the rod from X to Y.

11 $\varepsilon = \ell vB = 8.00 \times 4.00 \times 5.00 \times 10^{-5} = 1.60 \times 10^{-3}$ V or 1.60 mV

12



A change in the emf in S_1 produces a current in S_2. So no current flows in S_2 between $t = 1$ s and $t = 4$ s. An increase in emf at a constant rate ($t = 0$ to $t = 1$ s) would produce a constant current, and a decrease in emf at a lower rate ($t = 4$ to $t = 7$ s) would produce a lower current in the opposite direction. Either the graph shown or its inversion is correct.

13 $\frac{I_s}{I_p} = \frac{V_p}{V_s}$

$\frac{I_s}{3.00} = \frac{14.0}{42.0}$

$I_s = 1.00$ A

14 $\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$\frac{N_p}{30} = \frac{14.0}{42.0}$

$N_p = 10$ turns

15 A. The spikes in the voltage output occur when the input voltage rises and falls, i.e. when it changes.

16 **a** $V_{rms} = \frac{V_p}{\sqrt{2}} = \frac{25.0}{\sqrt{2}} = 18.0$ V

b $P_p = V_P I_P = 25.0 \times 15.0 = 375$ W

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17 $P_{rms} = V_{rms} \times I_{rms}$

$$P_{rms} = \frac{\left(\frac{1}{2}V_{p-p}\right)}{\sqrt{2}} \times \frac{\left(\frac{1}{2}I_{p-p}\right)}{\sqrt{2}} = \frac{V_{p-p} \times I_{p-p}}{8}$$

$$= \frac{40 \times 12}{8}$$

$P_{rms} = 60\,W$

Therefore option C is consistent with the specifications.

18 $\varepsilon_{max} = -2\pi NBA_{\perp}f$

$= -2\pi \times 500 \times 80.0 \times 10^{-3} \times 10.0 \times 10^{-4} \times 50.0$

$= 12.6\,V$

$\varepsilon_{max} = \frac{\varepsilon_{max}}{\sqrt{2}} = 8.91\,V$

19 Doubling the frequency doubles the rms emf, since the rate of change of flux is doubled.

20 Any two of:

- Using a DC power supply means that the voltage cannot be stepped up or down with transformers.
- There will be significant power loss along the 8 Ω power lines.
- There could be damage to any appliances operated in the shed that are designed to operate on 240 V AC and not on 240 V DC.

21 As the coil area is reduced, the flux into the page will decrease. To oppose this, the induced current will try to increase the flux again in the same direction. Using the right-hand grip rule the direction of the induced current will be clockwise.

22 AB and CD. Both the sides AB and CD cut across lines of flux as the coil rotates.

23 $P = VI$

$150 \times 10^3 = 10\,000 \times I$

$I = \frac{150 \times 10^3}{10\,000} = 15.0\,A$

24 Calculate the voltage drop:

$\Delta V = IR = 15.0 \times 2.00 = 30.0\,V$

Calculate the final voltage: Initial voltage – voltage drop

$\Delta V = 10000 - 30.0 = 9970\,V$

25 $P_{loss} = I^2R$

Using current calculated from Question 23, $I = 15.0\,A$, $P_{loss} = 15.0^2 \times 2.00 = 450\,W$

26 Without the first transformer, voltage in the transmission lines, $V = 1000\,V$.

Calculate I using $P = VI$

$I = \frac{P}{V} = \frac{150 \times 10^3}{1000} = 150\,A$

$P_{loss} = I^2R = 150^2 \times 2.00$

$= 45000\,W$

$P_{supplied} = 150\,kW - 45.0\,kW = 105\,kW$

This represents a 30% power loss—a bad idea!

27 Anticlockwise. Initially there is no flux through the coil. As the coil begins to rotate, the amount of flux increases and is directed to the left. To oppose this change, an induced magnetic field will be directed to the right. Using the right-hand grip rule this creates an anticlockwise current in the coil for the orientation shown in the diagram.

28 B. The power equation is $P = VI$ and the '2' indicates the secondary coil.

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29 **a** $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

$\frac{N_s}{800} = \frac{12.0}{240}$

$N_s = 40$ turns

b $\frac{I_p}{I_s} = \frac{N_s}{N_p}$

$\frac{I_p}{2.00} = \frac{40}{800}$

$I_p = 0.100$ A (rms current in primary)

$I_{peak} = I_{rms} \times \sqrt{2}$

$I_{peak} = 0.100 \times \sqrt{2}$

$I_{peak} = 0.141$ A

c $P_{rms} = V_{rms} I_{rms}$

$P_{rms} = 240 \times 0.100$

$P_{rms} = 24.0$ W

d A. A DC supply operates at a constant voltage, hence there is no changing flux through the secondary coil so no output voltage will be produced and the transformer will not operate.

30 $I = \frac{P}{V} = \frac{5.00 \times 10^6}{250 \times 10^3} = 2000$ A

$P_{loss} = I^2R = 2000^2 \times 10.0 = 4.00 \times 10^7$ W or 40.0 MW

31 B. A is incorrect because the *V* in the formula indicates the voltage drop in the transmission lines; it does not refer to the voltage being transmitted.

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Unit 3 Review

Section 1: Short response

1 From Newton's law of gravitation, the force due to gravity between two objects is calculated as:

$$F_g = \frac{Gm_1m_2}{r^2}$$
$$= \frac{(6.67\times10^{-11})\times(24.0)\times(903)}{0.670^2}$$
$$= 3.22\times10^{-6}\text{ N}$$

2 Kepler's third law implies that, for any two planets A and B:

$$\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$$
$$\therefore T_B^2 = \frac{T_A^2}{r_A^3}r_B^3$$
$$\therefore T_B = \sqrt{\frac{T_A^2}{r_A^3}r_B^3}$$

N.B. since our equation is just an equality of two ratios, it does not matter what units you use—as long as you are consistent. In this case it is easiest to use periods in days and radius in millions of kilometres.

$$T_{Jupiter} = \sqrt{\frac{(687)^2}{(227.9)^3}(778.5)^3}$$
$$= 4337$$
$$= 4340\text{ days}$$

3 **a** This problem can be solved by using the principle of conservation of mechanical energy. At the start, there is potential energy and no kinetic energy; at the end, there is kinetic energy but no potential energy. Hence:

$$mg\Delta h = \frac{1}{2}mv^2$$
$$\therefore 9.80\times3.50 = \frac{1}{2}v^2$$
$$\therefore v = \sqrt{2\times9.80\times3.50}$$
$$= 8.28\text{ m s}^{-1}$$

b The energy lost to friction will be exactly the difference between the initial and final mechanical energies, since this difference is due to the friction.

Initial mechanical energy:

$$E_i = mg\Delta h$$
$$= 21.0\times9.80\times3.50$$
$$= 7.20\times10^2\text{ J}$$

Final energy:

$$E_f = \frac{1}{2}mv^2$$
$$= \frac{1}{2}\times21.0\times5.00^2$$
$$= 2.625\times10^2\text{ J}$$

Hence the total energy lost to friction:

$$E_{tot} = E_i - E_f$$
$$= 7.20\times10^2 - 2.65\times10^2$$
$$= 4.58\times10^2\text{ J}$$

4 **a** All projectile trajectories (if friction is neglected) are symmetric. Hence when the cannonball reaches the same point above ground level as when it was fired, it will be going at the same speed, but downwards. So it is travelling 35.0 m s^{-1} at an angle of 40.0° down from horizontal.

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b Taking the lowest point as zero height, conservation of energy tells us that:

$$E_f = E_i$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mg\Delta h$$

$$v_f^2 = v_i^2 + 2g\Delta h$$

$$v_f = \sqrt{v_i^2 + 2g\Delta h}$$

$$= \sqrt{35.0^2 + 2 \times 9.80 \times (30.0 - 0)}$$

$$= 42.6\,\text{m s}^{-1}$$

5 The setup shown is in both translational and rotational equilibrium, hence all forces and torques sum to zero. Taking X as the pivot:

$$\Sigma M_{acw} = \Sigma M_{cw}$$

$$F_T r_\perp = F_w r_\perp$$

$$F_T r_\perp \sin 45° = mgr_\perp$$

$$F_T \times 0.350 \times \sin 45° = 0.100 \times 9.80 \times 0.650$$

$$F_T = 2.57\,\text{N}$$

6 $F_B = F_E$

$$qvB = Eq$$

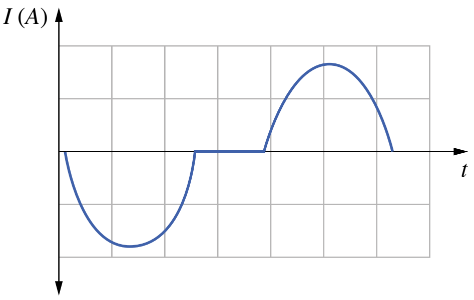
$$vB = \frac{\Delta V}{d}$$

$$d = \frac{\Delta V}{vB}$$

$$d = \frac{3.00 \times 10^3}{2.00 \times 10^7 \times 1.60 \times 10^{-3}}$$

$$d = 9.38 \times 10^{-2}\,\text{m}$$

7



There are three stages to the flight of the bar magnet that will influence the diagram: the point at which the magnet enters the coil, the time during which it is moving through the coil, and the time at which it is exiting the coil.

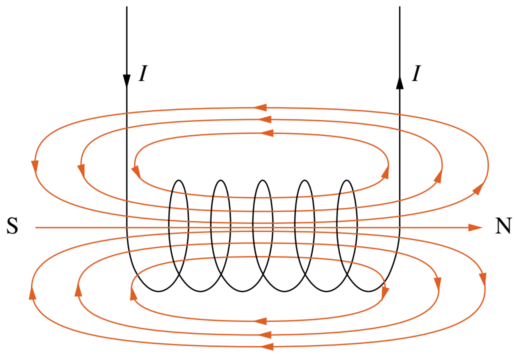
Lenz's law states that the direction of current induced by a changing magnetic field is such that the magnetic field of the induced current will oppose the change in flux.

When the magnet is entering the coil, there is an increased flux going down *into* the coil, and so current will be induced in order to create a magnetic flux going up out of the coil. By the right-hand grip rule, this will be anticlockwise when viewed from above.

When the magnet is inside the coil, the magnetic field will be approximately constant, and so there will be no change in the flux through the coil—hence no current.

Converse to the first stage, when the magnet is leaving the coil, there is a decrease in the magnetic flux in the downwards direction, and so the magnet will induce a current in order to increase the flux in the downwards direction. Using the right-hand grip rule this will be clockwise when viewed from above.

8 **a**



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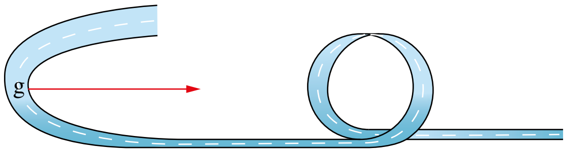
b A solenoid's magnetic field strength can be increased by increasing the current flowing through the coil, increasing the number of turns in the coil, or by placing a bar of iron inside the coil.

c According to the right-hand grip rule, with current running in opposite directions, the two magnetic fields will be in opposite directions (analogous to a bar magnet, the two solenoids will be as if they were opposite north and south poles next to each other). They will therefore attract each other.

Section 2: **Problem solving**

9 **a**



b By modifying the angle of the track such that 5.00 m s⁻¹ is the design speed, we can ensure that no friction is acting on the car either up or down the track. Hence:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\theta = \tan^{-1}\left(\frac{5.00^2}{2.50 \times 9.80}\right)$$

$$\theta = 45.6°$$

c At the design speed, the car is neither moving up nor down the track. The normal force can be resolved into its vertical and horizontal components. Since the forces up and down on the car are balanced, these forces can be equated:

$F_{Nv} = F_N \cos\theta$ and $F_g = mg$

$F_{Nv} = F_g$

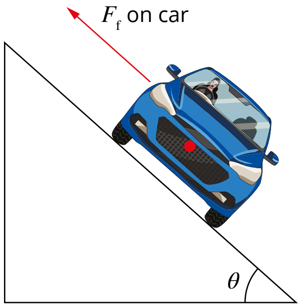
$F_N \cos\theta = mg$

$F_N = \frac{mg}{\cos\theta}$

$F_N = \frac{0.800 \times 9.80}{\cos 45.6°}$

$F_N = 11.2\,\text{N}$

d



e The critical speed at which the car is able to travel upside down in the loop is exactly when the normal force, or reaction force, acting on the car is zero.

$F_c = F_N + F_g$

$F_c = 0 + F_g$

$\frac{mv^2}{r} = mg$

$v^2 = gr$

$v = \sqrt{gr}$

$v = \sqrt{9.80 \times 1.50}$

$v = 3.83\,\text{m s}^{-1}$

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10 a A geostationary orbit is the orbit of a satellite around Earth for which the period of the orbit is exactly one day. In other words, a geostationary satellite is one that orbits the Earth at exactly the rate at which the Earth turns, and so it appears to be stationary in the sky.

b The period of a geostationary orbit will be 24 hours = 86 400 seconds, hence:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(86400)^2}{4\pi^2}}$$

$r = 4.22 \times 10^7$ m

$h = r + r_E$

$h = 4.22 \times 10^7 - 6.37 \times 10^6$

$h = 3.58 \times 10^7$ m above the surface of Earth

c $\frac{v^2}{r} = \frac{GM}{r^2}$

$$\therefore v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24}}{(20143+6371)\times10^3}}$$

$= 3.88 \times 10^3$ m s^{-1}

d $s = vt$

$s = 3.00 \times 10^8 \times 50 \times 10^{-9}$ m

$s = 15$ m

11 a The current is flowing from B to A, and so the force on this side of the coil will be directed upwards. The current flowing through the circuit is given by:

$$I = \frac{\Delta V}{R}$$

$$I = \frac{12.0}{2.00}$$

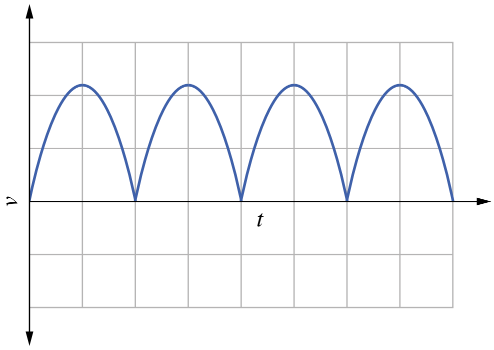
$I = 6.00$ A

$F = nI\ell B$

$F = 25 \times 6.00 \times 8.00 \times 10^{-2} \times 1.50 \times 10^{-3}$

$F = 1.80 \times 10^{-2}$ N upwards

b



c It takes 10.0 ms for the shaft to complete a quarter of a rotation. Using Faraday's law:

$$\text{emf} = -N\frac{\Delta\Phi}{t_{\frac{1}{4}}}$$

$$\text{emf} = -25\frac{(0)-(1.50\times10^{-3}\times(0.08)^2)}{10.0\times10^{-3}}$$

emf = 2.40×10^{-4} V

d Slip rings allow the coil to maintain constant contact with the circuit, and so will have current flowing in a constant direction. Considering the right-hand palm rule, in the beginning side AB will experience an upwards force, causing the coil to rotate clockwise. Once the coil has rotated by 180°, the side CD will be where AB previously was; however, the current will be flowing in the opposite direction since the coil has flipped. The right-hand grip rule tells us that CD will then experience a downwards force. The coil will then oscillate back and forth between each position until it likely reaches an equilibrium position turned halfway.

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12 **a** With more appliances on, the amount of current required would increase. An increase in current would increase the power loss in the transmission line as $P_{loss} = I^2R$. This power loss would cause a significant drop in potential across the transmission lines according to $\Delta V = IR$.

b Total current in the circuit at maximum power:

$P = \Delta VI$

$I = \frac{P}{\Delta V}$

$I = \frac{4.00 \times 10^3}{250}$

$I = 16.0\,\text{A}$

$\Delta V = IR$

$\Delta V = 16.0 \times 13.0$

$\Delta V = 208\,\text{V}$

$V_{house} = V - \Delta V$

$V_{house} = 250 - 208$

$V_{house} = 42.0\,\text{V}$

$P_{loss} = I^2R$

$P_{loss} = 16.0^2 \times 13.0$

$P_{loss} = 3328\,\text{W}$

$P_{house} = 4000 - 3328$

$P_{house} = 672\,\text{W}$

c At the generator end, a step-up transformer is needed, with a turns ratio of 1:20 to convert 250 V up to 5000 V. At the house end of the line, a step-down transformer is needed, with an approximate turns ratio of 20:1 to convert the approximately 5000 V down to 250 V again.

d $P_{in} = P_{out}$

$V_1I_1 = V_2I_2$

$I_2 = \frac{V_1I_1}{V_2}$

$I_2 = \frac{250 \times 16.0}{5000}$

$I_2 = 0.800\,\text{A}$

e $\Delta V = IR$

$\Delta V = 0.800 \times 13.0$

$\Delta V = 10.4\,\text{V}$

f $P_{loss} = I^2R$

$P_{loss} = 0.800^2 \times 13.0$

$P_{loss} = 8.32\,\text{W}$

g $V = V - \Delta V$

$V = 5000 - 10.4$

$V = 4989.6\,\text{V}$

h $P_{house} = P - P_{loss}$

$P_{house} = 4000 - 8.32$

$P_{house} = 3991.7\,\text{W}$

i Power losses in the system without transformers results in an efficiency of 16.8%, compared to an efficiency with transformers of 99.8%.

j With a higher transmission voltage, there is a corresponding decrease in current in the transmission lines. The power loss equation relates power loss to the square of the current ($P_{loss} = I^2R$) so any decrease in current has a significant effect on the power loss.

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Section 3: **Comprehension**

13 a The side-on view (right diagram) would be the more ideal view of the stellar system. Since the Doppler spectroscopy method relies on detecting the Doppler shift of the star in order to determine its velocity, the star must be moving either towards or away from Earth. Thus, a side view will be best because it has all of the star's velocity in the radial direction towards Earth. A top view (as shown in the left diagram) would give no information to astronomers about the velocity of the star.

b i In a general stellar system in which the plane of the system is not exactly side-on to Earth, some of the velocity of the star will be perpendicular to the line-of-sight of an astronomer. This velocity is undetectable to observers on Earth. Therefore, the radial velocity detected will only be one component of the total velocity of a star, meaning that its true velocity will be greater than this detected velocity. From the formula $\frac{M_{planet}}{v_{star}} = \frac{M_{star}}{v_{planet}}$, it can be seen that the mass of the planet, $M_{planet} = \frac{v_{star} M_{star}}{v_{planet}}$, is directly proportional to the velocity of the star. Since observations from Earth using this method provide only a lower bound for the star's velocity, they will only provide a lower bound for the planet's mass, with the true quantity determined by the star's true velocity.

ii In order to observe the transit of a planet in front of its local star, the star system must be side-on to observers on Earth. In light of the answers to the previous two questions, one would expect the measurement of the star's velocity to be close to its true velocity, and therefore for these mass calculations to be accurate.

c From Kepler's third law:

$$r^3 = \frac{GM_{star}}{4\pi^2} T_{star}^2$$

$$\therefore r = \sqrt[3]{\frac{GM_{star}}{4\pi^2} T_{star}^2}$$

$$= \sqrt[3]{\frac{6.67 \times 10^{-11} \times 1.59 \times 10^{29}}{4\pi^2} (4.05 \times 86400)^2}$$

$$r = 3.204 \times 10^9 \text{ m}$$

Using Newton's law of gravitation:

$$v_{planet} = \sqrt{\frac{GM_{star}}{r}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 1.59 \times 10^{29}}{3.204 \times 10^9}}$$

$$v_{planet} = 5.75 \times 10^4 \text{ m s}^{-1}$$

Finally, knowing the velocity of the planet, you can determine its mass:

$$\frac{M_{planet}}{v_{star}} = \frac{M_{star}}{v_{planet}}$$

$$\therefore M_{planet} = \frac{v_{star} M_{star}}{v_{planet}}$$

$$= \frac{1.59 \times 10^{29} \times 0.865}{5.75 \times 10^4}$$

$$M_{planet} = 2.39 \times 10^{24} \text{ kg}$$

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Chapter 7 Wave–particle duality and the quantum theory

Section 7.1 Properties of waves in two dimensions

Worked example: Try yourself 7.1.1

CALCULATING REFRACTIVE INDEX

The speed of light in crown glass (a type of glass used in optics) is $1.97 \times 10^8\,m\,s^{-1}$. Given that the speed of light in a vacuum is $3.00 \times 10^8\,m\,s^{-1}$, calculate the refractive index of crown glass.	
Thinking	**Working**
Recall the definition of refractive index.	$n = \frac{c}{v}$
Substitute the appropriate values into the formula and solve.	$n = \frac{3.00 \times 10^8}{1.97 \times 10^8} = \frac{3.00}{1.97} = 1.52$

Worked example: Try yourself 7.1.2

SPEED OF LIGHT CHANGES

A ray of light travels from water ($n = 1.33$) where it has a speed of $2.25 \times 10^8\,m\,s^{-1}$ into glass ($n = 1.85$). Calculate the speed of light in glass.	
Thinking	**Working**
Recall the formula.	$n_1v_1 = n_2v_2$
Substitute the appropriate values into the formula and solve.	$1.33 \times 2.25 \times 10^8 = 1.85 \times v_2$ $\therefore \frac{1.33 \times 2.25 \times 10^8}{1.85}$ $\therefore v_2 = 1.62 \times 10^8\,m\,s^{-1}$

Worked example: Try yourself 7.1.3

USING SNELL'S LAW

A ray of light in air strikes a piece of flint glass ($n = 1.62$) at an angle of incidence of 50° to the normal. Calculate the angle of refraction of the light in the glass.	
Thinking	**Working**
Recall Snell's law.	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Recall the refractive index of air.	$n_1 = 1.00$
Substitute the appropriate values into the formula to find a value for $\sin\theta_2$.	$1.00 \times \sin 50° = 1.62 \times \sin\theta_2$ $\sin\theta_2 = 0.4729$
Calculate the angle of refraction.	$\therefore \theta_2 = \sin^{-1} 0.4729 = 28.2°$

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Worked example: Try yourself 7.1.4

APPLYING SNELL'S LAW

Light travels from diamond into air. Describe the changes in the angle and the wavelength when the light enters the air. Use Table 7.1.1. Calculations are not necessary.	
Thinking	**Working**
Use Table 7.1.1 to find the velocity of light for the two substances.	$v_{diamond} = v_1 = 1.24 \times 10^8\,\text{m s}^{-1}$ $v_{air} = v_2 = 3 \times 10^8\,\text{m s}^{-1}$
Recall that velocity is proportional to the angle and the wavelength and compare the two velocities.	$v_1 < v_2$ then $\theta_1 < \theta_2$ $\lambda_1 < \lambda_2$
Describe the effect on the refracted wave.	The light speed is increasing, therefore the light is refracted way from the normal, as in Figure 7.1.10 on page 236. There will be a significant change in angle and a corresponding increase in wavelength.

Worked example: Try yourself 7.1.5

CALCULATING CRITICAL ANGLE

Calculate the critical angle for light passing from diamond into air.	
Thinking	**Working**
Recall the equation for critical angle.	$\sin\theta_c = \frac{n_2}{n_1}$
Substitute the refractive indexes of diamond and air into the formula.	$\sin\theta_c = \frac{1.00}{2.42} = 0.4132$
Solve for θ_c.	$\theta_c = \sin^{-1}0.4132 = 24.4°$

7.1 Review

1 **a** wave model
b wave model
c particle model

2 C. Newton's esteemed reputation meant that his theory was regarded as correct.

3 Therefore, the speed of light in seawater will be **less than** in pure water.

4 Recall the definition of refractive index: $n = \frac{c}{v}$
Rearrange to get

$$v = \frac{c}{n} = \frac{3.00 \times 10^8}{1.38} = 2.17 \times 10^8\,\text{m s}^{-1}$$

5 $n_1v_1 = n_2v_2$
$1.33 \times 2.25 \times 10^8 = n_2 \times 2.29 \times 10^8$
$n_2 = 1.31$

6 Recall Snell's law: $n_1\sin\theta_1 = n_2\sin\theta_2$
$1.33 \times \sin 44° = 1.60 \times \sin\theta_2$

$$\sin\theta_2 = \frac{1.33 \times \sin 44°}{1.60} = 0.5774$$

$$\theta_2 = \sin^{-1}0.5774 = 35.3°$$

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7 B, C. Total internal reflection occurs when the refractive index of the incident medium (n_1) is greater than the refractive index of the refracting medium (n_2).

8 D. Significant diffraction occurs when $\frac{\lambda}{w}$ is approximately 1 or greater. 700 nm = 0.0007 mm.

9 Polarisation is a phenomenon in which transverse waves are restricted in their direction of vibration. Polarisation can only occur in transverse waves and cannot occur in longitudinal waves. Since light can be polarised, it must be a transverse wave.

Section 7.2 Interference: Further evidence for the wave model of light

Worked example: Try yourself 7.2.1

CALCULATING WAVELENGTH FROM FRINGE SEPARATION

Green laser light is directed through a pair of thin slits that are 25 μm apart. The slits are 1.5 m from a screen on which bright fringes are 3.3 cm apart. Use this information to calculate the wavelength of green light in nm.	
Thinking	**Working**
Recall the equation for fringe separation.	$\Delta x = \frac{\lambda L}{d}$
Transpose the equation to make λ the subject.	$\lambda = \frac{\Delta x d}{L}$
Substitute values into the equation and solve.	$\lambda = \frac{0.033 \times 25 \times 10^{-6}}{1.5} = 5.5 \times 10^{-7}$ m
Express your answer using the unit specified.	λ = 550 nm

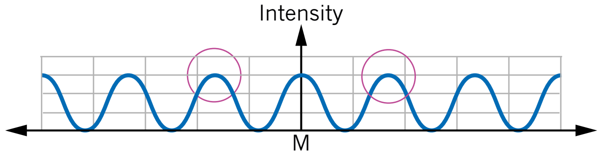
7.2 Review

1 D. Light passed through the double slits to hit the screen. Young's double-slit experiment produced an interference pattern of alternating bright and dark lines on the screen.

2 C and D. As laser light is monochromatic and coherent, it is more likely to produce the interference pattern expected in Young's experiment.

3 A and D. When crests meet troughs, the addition of these out-of-phase waves means that they cancel to form a node.

4 The central antinode occurs where both waves have travelled the same distance, i.e. the path difference is 0. The next antinodes on either side occur when the path difference is 1λ.



5 Up until Young's experiment, most scientists supported a particle or 'corpuscular' model of light. Young's experiment demonstrated interference patterns, which are characteristic of waves. This led to scientists abandoning the particle theory and supporting a wave model of light.

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6 Recall the equation for fringe separation: $\Delta x = \frac{\lambda L}{d}$

a increase
b decrease
c increase

7 $\text{pd} = \left(n - \frac{1}{2}\right)\lambda$

For the fifth dark fringe, $n = 5$

$\text{pd} = \left(5 - \frac{1}{2}\right)\lambda$

Therefore, the fifth dark fringe occurs where the path difference is $4.5\lambda = 4.5 \times 580\,\text{nm} = 2610\,\text{nm}$ or $2.61 \times 10^{-6}\,\text{m}$.

8 Constructive interference occurs when the path difference is a whole number multiple of the wavelength. Destructive interference occurs when the path difference is an odd number multiple of half the wavelength.

a destructive
b constructive
c destructive

9 $\text{pd} = n\lambda$

For the second bright fringe, $n = 2$

$\text{pd} = 2\lambda$

Therefore, the second bright fringe occurs where the path difference is $2 \times 700 = 1400\,\text{nm}$.

10 $\Delta x = \frac{\lambda L}{d}$

$\lambda = \frac{\Delta x d}{L}$

$= \frac{0.037 \times 40 \times 10^{-6}}{3.25}$

$= 4.55 \times 10^{-7}\,\text{m}$

$= 455\,\text{nm}$

Section 7.3 Electromagnetic waves

Worked example: Try yourself 7.3.1

USING THE WAVE EQUATION FOR LIGHT

A particular colour of red light has a wavelength of 600 nm. Calculate the frequency of this colour.	
Thinking	**Working**
Recall the wave equation for light.	$c = f\lambda$
Transpose the equation to make frequency the subject.	$f = \frac{c}{\lambda}$
Substitute in values to determine the frequency of this wavelength of light.	$f = \frac{3.0 \times 10^{8}}{600 \times 10^{-9}}$ $= 5.0 \times 10^{14}\,\text{Hz}$

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7.3 Review

1 B. Mechanical waves require a medium whereas light waves can travel through a vacuum.

2 D. Light is electromagnetic radiation that is composed of changing electric and magnetic fields. Electric and magnetic waves oscillate at 90° to each other, so in an electromagnetic wave the changing electric and magnetic fields are orientated perpendicular to each other.

3 D. Electromagnetic radiation with a wavelength of 200 nm would be classified as ultraviolet light since this part of the spectrum has a shorter wavelength than visible light.

4 From shortest to longest wavelength: X-rays, visible light, infrared radiation, FM radio waves.

5 Use $c = f\lambda$

Transpose to make frequency the subject.

a red, $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{656 \times 10^{-9}} = 4.57 \times 10^{14}$ Hz

b yellow, $f = 5.09 \times 10^{14}$ Hz

c blue, $f = 6.17 \times 10^{14}$ Hz

d violet, $f = 7.56 \times 10^{14}$ Hz

6 $\frac{3 \times 10^8 - 299792458}{299792458} \times 100\% = 0.07\%$

7 Use $c = f\lambda$

Transpose to make wavelength the subject.

$$\lambda = \frac{c}{f}$$
$$= \frac{3 \times 10^8}{6.0 \times 10^{14}}$$
$$= 5 \times 10^{-7}$$
$$= 500\ \text{nm}$$

8 Use $c = f\lambda$

Transpose to make wavelength the subject.

$$\lambda = \frac{c}{f}$$
$$= \frac{3 \times 10^8}{7.0 \times 10^7}$$
$$= 4.2857$$
$$= 4.3\ \text{m}$$

9 Use $c = f\lambda$

Transpose to make frequency the subject.

$$f = \frac{c}{\lambda}$$
$$= \frac{3 \times 10^8}{200 \times 10^{-12}}$$
$$= 1.5 \times 10^{18}\ \text{Hz}$$

10 Frequency of radiation produced by a microwave oven is 2.45 GHz.

Use $c = f\lambda$

Transpose to make wavelength the subject.

$$\lambda = \frac{c}{f}$$
$$= \frac{3 \times 10^8}{2.45 \times 10^9}$$
$$= 0.122\ \text{m}$$

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Section 7.4 Light quanta: Blackbody radiation and the photoelectric effect

Worked example: Try yourself 7.4.1

USING PLANCK'S EQUATION

Calculate the energy in joules of a quantum of infrared radiation that has a frequency of 3.6×10^{14} Hz.	
Thinking	**Working**
Recall Planck's equation.	$E = hf$
Substitute in the appropriate values to solve.	$E = 6.63 \times 10^{-34} \times 3.6 \times 10^{14}$ $= 2.4 \times 10^{-19}$ J

Worked example: Try yourself 7.4.2

CONVERTING TO ELECTRON-VOLTS

A quantum of light has 2.4×10^{-19} J. Convert this energy to electron-volts.	
Thinking	**Working**
Recall the conversion rate for joules to electron-volts.	1 eV = 1.6×10^{-19} J
Divide the value expressed in joules by 1.6×10^{-19} J eV^{-1} to convert to electron-volts.	$\frac{2.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.5$ eV

Worked example: Try yourself 7.4.3

CALCULATING THE WORK FUNCTION OF A METAL

Calculate the work function (in J and eV) for gold, which has a threshold frequency of 1.2×10^{15} Hz.	
Thinking	**Working**
Recall the formula for work function.	$\phi = hf_0$
Substitute the threshold frequency of the metal into this equation.	$\phi = 6.63 \times 10^{-34} \times 1.2 \times 10^{15}$ $= 8.0 \times 10^{-19}$ J
Convert this energy from J to eV.	$\phi = \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 5.0$ eV

Worked example: Try yourself 7.4.4

CALCULATING THE KINETIC ENERGY OF PHOTOELECTRONS

Calculate the kinetic energy (in eV) of the photoelectrons emitted from lead by ultraviolet light with a frequency of 1.50×10^{15} Hz. The work function of lead is 4.14 eV.	
Thinking	**Working**
Recall Einstein's photoelectric equation.	$E_{k\,max} = hf - \phi$
Substitute values into this equation.	$hf = \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-19}} \times 1.50 \times 10^{15} = 6.22$ eV $E_{k\,max} = 6.22 - 4.14$ $= 2.08$ eV

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7.4 Review

1 **a** False
b False
c True
d True. The blackbody emits a range of wavelengths, depending on temperature.
e False. The radiation spectrum depends only on temperature.

2 1 Molecules vibrate at fixed frequencies or energies.
2 To change energy levels, a molecule needs to absorb or emit a photon of exactly the energy difference between two levels.

3 **a** $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^{8} \div (656 \times 10^{-9}) = 3.03 \times 10^{-19}\,\text{J}$

b $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^{8} \div (589 \times 10^{-9}) = 3.38 \times 10^{-19}\,\text{J}$

c $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^{8} \div (486 \times 10^{-9}) = 4.09 \times 10^{-19}\,\text{J}$

d $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^{8} \div (397 \times 10^{-9}) = 5.01 \times 10^{-19}\,\text{J}$

4 In the photoelectric effect, a metal surface may become positively charged if the energy of the photons of light shining on it is greater than the work function of the metal and causes electrons to be released.

5 **a** True
b False. When light sources of the same intensity but different frequencies are used, the higher frequency light has a higher stopping voltage, but it produces the same maximum current as the lower frequency.
c True

6 **a** $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.0 \times 10^{15} = 4.1\,\text{eV}$
b $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.1 \times 10^{15} = 4.6\,\text{eV}$
c $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} = 6.2\,\text{eV}$

7 D. The threshold frequency is:

$$f_0 = \frac{\phi}{h}$$
$$= \frac{3.66}{4.14 \times 10^{-15}}$$
$$= 8.84 \times 10^{14}\,\text{Hz}$$

In order to release photoelectrons, the light must have a frequency higher than the threshold frequency. Therefore, 9.0×10^{14} Hz is the only frequency that will release photoelectrons.

8 $E_{\text{k max}} = 4.14 \times 10^{-15} \times 9.0 \times 10^{14} - 3.66$
$= 0.066\,\text{eV}$

9 $$E = \frac{hc}{\lambda}$$
$$= \frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{475 \times 10^{-9}} = 2.61\,\text{eV}$$
$E_{\text{k max}} = 2.61 - 2.36$
$= 0.25\,\text{eV}$

10 C and D.

$$\phi = hf_0 = \frac{hc}{\lambda_0}$$
$$\lambda_0 = \frac{hc}{\phi} = \frac{4.14 \times 10^{-15} \times 3.00 \times 10^{8}}{1.81}$$
$$= 6.86 \times 10^{-7}\,\text{m}$$
$$= 686\,\text{nm}$$

Photons with wavelengths shorter than the threshold wavelength—i.e. violet light and ultraviolet radiation—will cause photoelectrons to be emitted.

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11 a True

b False. The stopping voltage is reached when the photocurrent is reduced to zero.

c True

d True

12 $E_{k\ max} = hf - \phi$

$= \frac{hc}{\lambda} - \phi$

$0.80 = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{500 \times 10^{-9}} - \phi$

$\phi = 2.48 - 0.80$

$= 1.68\,\text{eV}$

Section 7.5 Atomic spectra

Worked example: Try yourself 7.5.1

USING THE BOHR MODEL OF THE HYDROGEN ATOM

Calculate the wavelength (in nm) of the photon produced when an electron drops from the $n = 3$ energy level of the hydrogen atom to the $n = 1$ energy level. Identify the spectral series to which this line belongs. Use Figure 7.5.6 to calculate your answer.	
Thinking	**Working**
Identify the energy of the relevant energy levels of the hydrogen atom.	$E_3 = -1.50\,\text{eV}$ $E_1 = -13.6\,\text{eV}$
Calculate the change in energy.	$\Delta E = E_3 - E_1$ $= -1.5 - (-13.6)$ $= 12.1\,\text{eV}$
Calculate the wavelength of the photon with this amount of energy. Either use $h = \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-19}}$ eV or convert ΔE in eV to joules i.e. $\Delta E = 12.1 \times 1.6 \times 10^{-19}\,\text{J}$	$\Delta E = \frac{hc}{\lambda}$ $\therefore \lambda = \frac{hc}{E}$ $= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 12.1}$ $= 1.03 \times 10^{-7}\,\text{m}$ $= 103\,\text{nm}$
Identify the spectral series.	The electron drops down to the $n = 1$ energy level. Therefore, the photon must be in the Lyman series.

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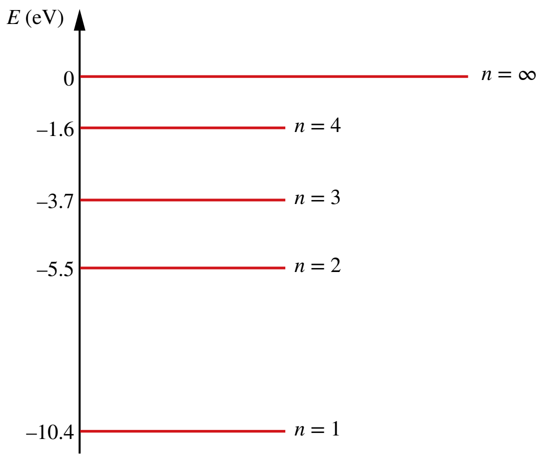
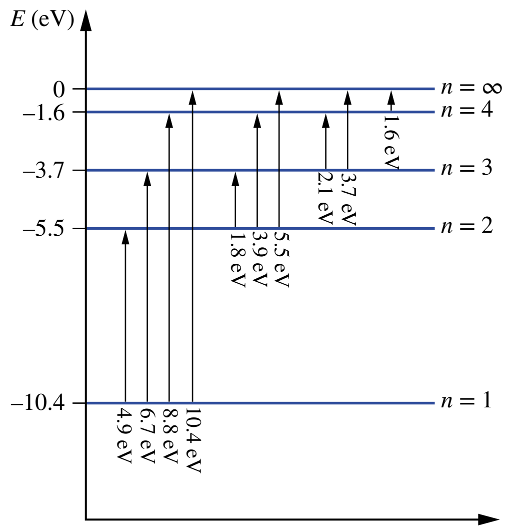
Worked example: Try yourself 7.5.2

ABSORPTION OF PHOTONS

Some of the energy levels for atomic mercury are shown in the diagram below.



Light with photon energies 6.7, 9.0 and 11.0 eV is incident on some mercury gas. What could happen to as a result of the incident light?

Thinking	Working
Check whether the energy of each photon corresponds to any differences between energy levels by determining the difference in energy between each pair of levels.	
Compare the energy of the photons with the energies determined and comment on the possible outcomes.	A photon of 6.7 eV corresponds to the energy required to promote an electron from the ground state to the second excited state (n = 1 to n = 3). The photon may be absorbed. A photon of 5.0 eV cannot be absorbed. A photon of 11.0 eV may ionise the mercury atom. The ejected electron will leave the atom with 0.6 eV of kinetic energy.

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Worked example: Try yourself 7.5.3

BAND GAP OF LEDS

A GaN LED has a threshold voltage of 3.40 V. Determine the band gap, the wavelength of emission and expected colour.	
Thinking	**Working**
The threshold voltage is related to the band gap by the relationship: $eV_{th} = E_g$	$V_{th} = 3.40$ V so $eV_{th} = 3.40$ eV and $E_g = 3.40$ eV
Rearrange $\frac{hc}{\lambda} = E_g$.	$\lambda = \frac{hc}{E_g}$ $= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 3.40}$ $= 3.66 \times 10^{-9}$ $= 366$ nm
Use Table 7.1.3 (page 241).	The wavelength would be in the ultraviolet.

7.5 Review

1 The electrons in a sample become excited when the substance is heated, an electric current flows through it or it is excited by photons or electrons. As the electrons return to their ground state, a photon is emitted.

2 $\Delta E = hf$

$= 6.63 \times 10^{-34} \times 6.0 \times 10^{14}$

$= 4.0 \times 10^{-19}$ J

3 $\Delta E = \frac{hc}{\lambda}$

$\therefore \lambda = \frac{hc}{\Delta E}$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{0.42}$

$= 2.96 \times 10^{-6}$ m $= 3.0 \times 10^{-6}$ m

4 **a** light-emitting diode

b light amplification by stimulated emission of radiation

5 $\Delta E = \frac{hc}{\lambda}$

$\therefore \lambda = \frac{hc}{\Delta E}$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{1.84}$

$= 6.75 \times 10^{-7}$ m

$= 675$ nm

6 $\Delta E = E_4 - E_1$

$= -0.85 - (-13.6)$

$= 12.75$ eV

7 $\Delta E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{\Delta E}$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{12.75}$

$= 9.74 \times 10^{-8}$ m or 97.4 nm

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8 It could not explain high-energy orbits of multi-electron atoms, the continuous emission spectrum of solids, or the two close spectral lines in hydrogen that are revealed at high resolution.

9 $\Delta E = \frac{hc}{\lambda} = E_5 - E_2$

$\Delta E = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{434 \times 10^{-9}} = E_5 - (-3.4)$

$E_5 = 2.86 - 3.4$

$= -0.54\,\text{eV}$

10 The energy difference between $n = 1$ and $n = 3$ is 6.7 eV.

An electron from the electron beam will promote the atomic electron from $n = 1$ to $n = 3$ and lose 6.7 eV. It will then exit the atom with an energy of 0.3 eV.

A photon has exactly 7 eV of energy and needs to give up all of its energy, therefore it cannot promote the electron and will pass straight through.

Section 7.6 The quantum nature of light and matter

Worked example: Try yourself 7.6.1

CALCULATING THE DE BROGLIE WAVELENGTH

Calculate the de Broglie wavelength of a proton travelling at $7.0 \times 10^{5}\,\text{m s}^{-1}$. The mass of a proton is 1.67×10^{-27} kg.	
Thinking	**Working**
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 7.0 \times 10^{5}}$ $= 5.7 \times 10^{-13}\,\text{m}$

Worked example: Try yourself 7.6.2

CALCULATING THE DE BROGLIE WAVELENGTH OF A MACROSCOPIC OBJECT

Calculate the de Broglie wavelength of a person with mass of 66 kg running at $36\,\text{km h}^{-1}$.	
Thinking	**Working**
Convert velocity to SI units.	$v = 36 \div 3.6 = 10\,\text{m s}^{-1}$
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{66 \times 10}$ $= 1.0 \times 10^{-36}\,\text{m}$

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Worked example: Try yourself 7.6.3

WAVELENGTH OF ELECTRONS FROM AN ELECTRON GUN

Find the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 50 V. The mass of an electron is 9.11×10^{-31} kg and the magnitude of the charge on an electron is 1.6×10^{-19} C.	
Thinking	**Working**
Calculate the kinetic energy of the electron from the work done on it by the electric potential.	$W = qV$ $= 1.6 \times 10^{-19} \times 50$ $= 8.0 \times 10^{-18}$ J
Calculate the velocity of the electron.	$E_k = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2 \times E_k}{m}}$ $= \sqrt{\frac{2 \times 8.0 \times 10^{-18}}{9.11 \times 10^{-31}}}$ $= 4.2 \times 10^6$ m s^{-1}
Use de Broglie's equation to calculate the wavelength of the electron.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.2 \times 10^6}$ $= 1.7 \times 10^{-10}$ m $= 0.17$ nm

Worked example: Try yourself 7.6.4

CALCULATING PHOTON MOMENTUM

Calculate the momentum of a photon of blue light with a wavelength of 450 nm.	
Thinking	**Working**
Convert 450 nm to m.	450 nm = 450×10^{-9} m
Transpose de Broglie's equation to make momentum (*p*) the subject.	$\lambda = \frac{h}{p}$ $p = \frac{h}{\lambda}$
Substitute in values and solve for *p*.	$p = \frac{6.63 \times 10^{-34}}{450 \times 10^{-9}}$ $= 1.47 \times 10^{-27}$ kg m s^{-1}

7.6 Review

1 $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.0 \times 10^6}$

$= 7.3 \times 10^{-10}$ m

2 $\lambda = \frac{h}{mv}$

$4.0 \times 10^{-9} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times v}$

$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.0 \times 10^{-9}}$

$= 1.8 \times 10^5$ m s^{-1}

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3 B. Wave behaviour of matter is linked to the mass and the velocity (i.e. momentum) of the matter. So only moving particles exhibit wave behaviour.

4 **a** $\lambda = \frac{c}{f} = \frac{3.0 \times 10^{8}}{8.6 \times 10^{18}} = 3.5 \times 10^{-11}\,\text{m}$

b $\lambda = \frac{h}{mv}$

$3.5 \times 10^{-11} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times v}$

$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.5 \times 10^{-11}} = 2.1 \times 10^{7}\,\text{m s}^{-1}$

5 The wavelength of a cricket ball is so small that its wave-like behaviour could not be seen by a cricket player.

6 $\lambda = \frac{hc}{E}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{6.63 \times 10^{-14}}$

$= 3.0 \times 10^{-12}\,\text{m}$

Speed of the proton to exhibit this wavelength:

$v = \frac{h}{m\lambda}$

$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.0 \times 10^{-12}}$

$= 1.32 \times 10^{5}\,\text{m s}^{-1}$

7 $W = qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$

$\lambda = \frac{h}{mv}$

$= \frac{h}{m\frac{\sqrt{2qV}}{\sqrt{m}}}$

$\lambda = \frac{h}{\sqrt{2qVm}}$

8 $\lambda = \frac{h}{mv}$

$\lambda mv = h$

$mv = \frac{h}{\lambda}$

$p = \frac{h}{\lambda}$

9 An electron microscope can resolve images in finer detail than an optical microscope because a high-speed electron has a shorter wavelength than a light wave.

10 De Broglie proposed a model in which electrons were viewed as matter waves with wavelengths that formed standing waves within an atomic orbit circumference. A bowed violin string forms standing waves between the bridge of the violin and the violinist's finger.

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CHAPTER 7 REVIEW

1 A. This shows the bending of the edges of the waves as they pass through a gap.

2 Since $\Delta x = \frac{\lambda L}{d}$, the diffraction pattern would spread out more when the light is changed from blue to green.

The green light (λ = 525 nm) has a longer wavelength than blue light (λ = 460 nm). This longer wavelength results in more widely spaced fringes and a wider overall pattern.

3 D. Polarisation is a phenomenon in which transverse waves are restricted in their direction of vibration. Polarisation can only occur in transverse waves; it cannot occur in longitudinal waves. Light can be polarised, so it must be a transverse wave.

4 Both snow and water reflect light. This reflected light is known as glare. The light reflected from water and snow is partially polarised. Snowboarders and sailors would benefit from wearing polarising sunglasses as these will absorb the polarised glare from the snow or water respectively.

5 $c = f\lambda$

$= 4.5 \times 10^{14} \times 500 \times 10^{-9}$

$= 2.25 \times 10^{8}\ \text{m s}^{-1}$

6 As light travels from quartz (n = 1.46) to water (n = 1.33), its speed **increases**, which causes it to refract **away from** the normal.

7 A: incident ray

B: normal

C: reflected ray

D: boundary between media

E: refracted ray

8 $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$

$= \frac{1.00 \times \sin 43°}{\sin 28.5°}$

$= 1.429$

Since $n = \frac{c}{v}$

$v = \frac{c}{n}$

$= \frac{3.00 \times 10^{8}}{1.429}$

$= 2.1 \times 10^{8}\ \text{m s}^{-1}$

9 Use Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Angle a

$1.00 \times \sin 40° = 1.50 \times \sin a$

$\sin a = \frac{1.00 \times \sin 40°}{1.5}$

$= 0.4285$

$a = \sin^{-1}(0.4285) = 25.4°$

Angle b

Since a and b are corresponding angles, $a = b = 25.4°$

Angle c

$1.50 \times \sin 25.4° = 1.33 \times \sin c$

$\sin c = \frac{1.50 \times \sin 25.4°}{1.33}$

$= 0.4837$

$c = \sin^{-1} 0.4837 = 28.9°$

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10 **a** The angle of incidence is measured with respect to the normal, which is drawn at a right angle to the glass–air boundary.

$\theta_i = 90 - 58.0 = 32.0°$

b $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.52 \times \sin 32° = 1.00 \times \sin\theta_2$

$\sin\theta_2 = \frac{1.52 \times \sin 32°}{1.00}$

$= 0.8055$

$\theta_2 = \sin^{-1} 0.8055 = 53.7°$

c $\Delta\theta = \theta_2 - \theta_1$

$= 53.7 - 32$

$= 21.7°$

d $v = \frac{c}{n}$

$= \frac{3 \times 10^8}{1.52}$

$= 1.97 \times 10^8\,\text{m s}^{-1}$

11 **a** Red light

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \times \sin 30° = 1.50 \times \sin\theta_{red}$

$\sin\theta_{red} = \frac{n_1 \sin\theta_1}{n_2}$

$= \frac{1.00 \times \sin 30°}{1.50}$

$= 0.3333$

$\theta_2 = \sin^{-1} 0.3333 = 19.5°$

b Violet light

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \times \sin 30° = 1.53 \times \sin\theta_{violet}$

$\sin\theta_{violet} = \frac{n_1 \sin\theta_1}{n_2}$

$= \frac{1.00 \times \sin 30°}{1.53}$

$= 0.3268$

$\theta_2 = \sin^{-1} 0.3268 = 19.1°$

c $\Delta\theta = \theta_2 - \theta_1 = 19.5 - 19.1 = 0.4°$

d $v = \frac{c}{n} = \frac{3 \times 10^8}{1.53} = 1.96 \times 10^8\,\text{m s}^{-1}$

12 $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

a $\theta_c = \sin^{-1}\left(\frac{1.00}{1.31}\right) = 49.8°$

b $\theta_c = \sin^{-1}\left(\frac{1.00}{1.54}\right) = 40.5°$

c $\theta_c = \sin^{-1}\left(\frac{1.00}{2.16}\right) = 27.6°$

13 B, D, A, C. The bigger the difference in speed or refractive indices, the bigger the angle of deviation. The air–water boundary has the smallest difference in speed so it will produce the smallest angle of deviation. The air–diamond boundary has the biggest difference in speed so it will produce the biggest angle of deviation.

14 **a** $\Delta x = \frac{\lambda L}{d}$

$\lambda = \frac{\Delta x d}{L}$

$= \frac{0.031 \times 75 \times 10^{-6}}{4.0}$

$= 5.81 \times 10^{-7}$

$= 581\,\text{nm}$

b 581 nm is closest to yellow (according to Table 7.1.3)

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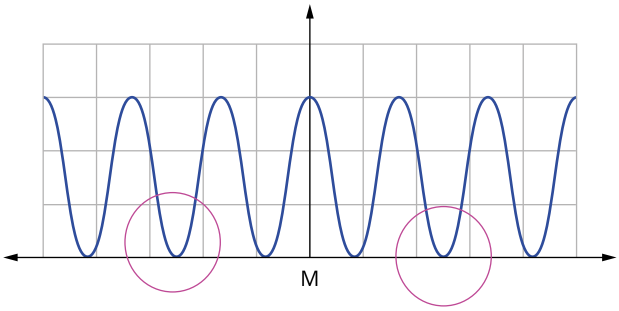
15 A path difference of $1\frac{1}{2}\lambda$ corresponds to the second dark band on each side of the central maximum at M.



16 In order of decreasing wavelength: radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays

17 **a** microwaves

b infrared waves

c X-rays, radio waves (in MRI), infrared radiation, visible light, UV and gamma radiation

18 Since $c = f\lambda$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{612 \times 10^3}$$

$$= 490\,\text{m}$$

19 Young performed his famous experiment in 1803, in which he observed an interference pattern for light. Young shone monochromatic light on a pair of narrow slits. Light passed through the slits and formed a pattern of bright and dark lines/fringes/bands on a screen. Young compared this to interference patterns he had observed, and he identified that these lines corresponded to regions of constructive and destructive interference. This could only be explained by considering light to be a wave.

20 A microwave oven is tuned to produce electromagnetic waves with a frequency of 2.45 GHz. This is the resonant frequency of water molecules. When food is bombarded with radiation at this frequency, the water molecules within the food start to vibrate. The energy of the water molecules is then transferred to the rest of the food, heating it up.

21 $E = hf = 4.14 \times 10^{-15} \times 6.0 \times 10^{14} = 2.5\,\text{eV}$

22 $E = 5.0 \times 1.6 \times 10^{-19}\,\text{J} = 8.0 \times 10^{-19}\,\text{J}$

23 photoelectrons

24 $\phi = hf_0$

$$f_0 = \frac{\phi}{h}$$

$$= \frac{5.0}{4.14 \times 10^{-15}}$$

$$= 1.2 \times 10^{15}\,\text{Hz}$$

25 $\phi = hf_0$

$$= 4.14 \times 10^{-15} \times 1.5 \times 10^{15}$$

$$= 6.2\,\text{eV}$$

$$E_{k\,max} = 4.14 \times 10^{-15} \times 2.2 \times 10^{15} - 6.2$$

$$= 2.9\,\text{eV}$$

26 The stopping voltage is equivalent to the maximum kinetic energy of the photoelectrons, so $E_{k\,max} = 1.95\,\text{eV}$.

27 The work function is given by the *y*-intercept of the $E_{k\,max}$ versus frequency graph. Approximate values are Rb = 2.1 eV, Sr = 2.5 eV, Mg = 3.4 eV, W = 4.5 eV.

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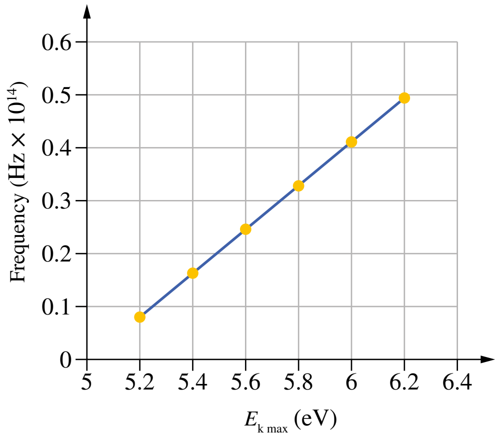
28 a



b gradient $= h = \frac{\text{rise}}{\text{run}}$

$= \frac{0.494 - 0.080}{6.20 \times 10^{14} - 5.20 \times 10^{14}}$

$= \frac{0.414}{1.00 \times 10^{14}}$

$= 4.1 \times 10^{-15}$ eV s

Note, in an experiment where the line of best fit does not lie exactly on the data, the gradient is taken from the line.

c The x-intercept on the graph is 5.0×10^{14} Hz.

d No. The frequency of red light is below the threshold frequency for rubidium.

Frequency of the red light:

$f = \frac{c}{\lambda}$

$= \frac{3.00 \times 10^{8}}{680 \times 10^{-9}}$

$= 4.41 \times 10^{14}$ Hz

This is less than the threshold frequency of 5.1×10^{14} Hz, so no photoelectrons will be emitted.

29 a $E = \frac{hc}{\lambda}$

$= \frac{4.14 \times 10^{-15} \times 3.00 \times 10^{8}}{260 \times 10^{-12}}$

$= 4777$ eV

$= 4.78$ keV

b The electrons have a de Broglie wavelength that is similar to the wavelength of the X-rays. This is evidence for the dual nature of light and matter.

c $p = \frac{h}{\lambda}$

$= \frac{6.63 \times 10^{-34}}{260 \times 10^{-12}}$

$= 2.55 \times 10^{-24}$

$= 2.6 \times 10^{-24}$ kg m s^{-1}

30 a The detector observed a sequence of maximum and minimum intensities.

b As the electron beam is diffracted, the electrons are exhibiting wave-like behaviour. Electrons are not light but, like light, a beam of electrons can be diffracted.

31 Energy levels in an atom cannot assume a continuous range of values but are restricted to certain discrete values, i.e. the levels are quantised.

32 $\Delta E = E_3 - E_1$

$= -1.5 - (-13.6)$

$= 12.1$ eV

$\Delta E = hf$

$f = \frac{\Delta E}{h}$

$= 12.1 \div (4.14 \times 10^{-15})$

$= 2.92 \times 10^{15}$ Hz

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33 Bohr's work on the hydrogen atom convinced many scientists that a particle model was needed to explain the way light behaves in certain situations. It built significantly on the work of Planck and Einstein.

34 The emission spectrum of hydrogen appears as a series of coloured lines. The absorption spectrum of hydrogen appears as a full visible spectrum with a number of dark lines. The colours missing from the absorption spectrum match the colours that are visible in the emission spectrum.

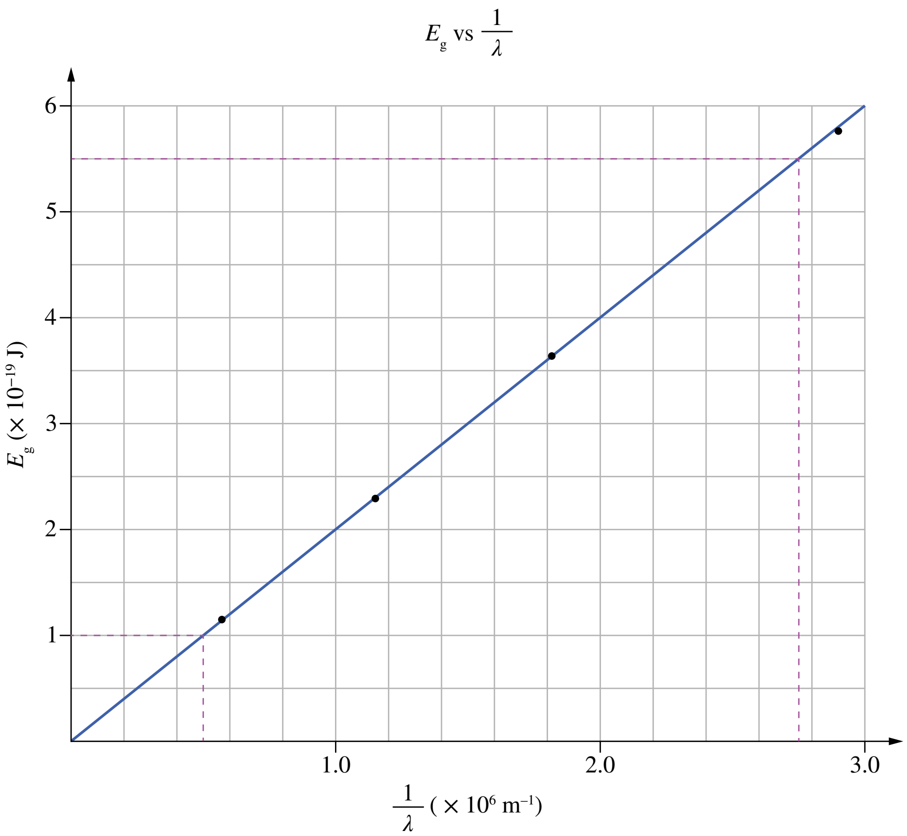
35 As the filament heats up, the free electrons in the tungsten atoms collide, accelerate and emit photons. A wide range of photon wavelengths are emitted due to a wide range of different collisions (some weak, some strong).

36 $eV_{th} = E_g$; E_g is stated in the table in J, i.e. E_g (J) = eV_{th} (eV) × 1.6 ×10^{-19}

And $E_g = \frac{hc}{\lambda}$; therefore, plotting E_g on the *y*-axis (in joules) and $\frac{1}{\lambda}$ (in m^{-1}) on the *y*-axis will give a gradient of *hc*, in SI units.

Semiconductor	Threshold voltage (V)	λ (nm)	E_g (J)	$\frac{1}{\lambda}$ (m^{-1})
GaSb	0.68	1828	1.09×10^{-19}	0.547×10^{6}
GaAs	1.43	869	2.29×10^{-19}	1.15×10^{6}
GaP	2.25	553	3.6×10^{-19}	1.81×10^{6}
ZnS	3.6	345	5.76×10^{-19}	2.90×10^{6}

The gradient = *hc*.



From the data, the gradient = $\frac{5.5 \times 10^{-19} - 1.0 \times 10^{-19}}{2.75 \times 10^{6} - 0.50 \times 10^{6}} = 2.0 \times 10^{-25}$

Therefore $h = \frac{\text{gradient}}{c} = \frac{2 \times 10^{-25}}{3 \times 10^{8}} = 6.7 \times 10^{-34}$

This is close to the accepted value of 6.63×10^{-34} J.

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37 Electron: $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 7.5 \times 10^{6}}$

$= 9.7 \times 10^{-11}\,\text{m}$

Blue light: $\lambda = 470 \times 10^{-9} = 4.7 \times 10^{-7}\,\text{m}$

X-ray: $c = f\lambda$ so $\lambda = \frac{c}{f} = \frac{3.0 \times 10^{8}}{5 \times 10^{17}} = 6 \times 10^{-10}\,\text{m}$

Proton: $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.7 \times 10^{-21}} = 3.9 \times 10^{-13}\,\text{m}$

$\therefore$ B, blue light, has the longest wavelength.

38 $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{0.040 \times 1.0 \times 10^{3}}$

$= 1.658 \times 10^{-35}$

$= 1.7 \times 10^{-35}\,\text{m}$

39 No—the wavelength is much smaller than the size of everyday objects.

The wavelength of the bullet travelling at $1.0 \times 10^{3}\,\text{m}\,\text{s}^{-1}$ is many times smaller than the radius of an atom. Significant diffraction only occurs when wavelength and gap (or object) size are approximately equal, i.e. when $\lambda \geq w$.

40 For the product of the uncertainty in position and the uncertainty in momentum to remain constant, as the uncertainty in position is decreased, the uncertainty in momentum would increase.

41 It is likely that the photon would knock the electron off course and hence the electron's position would be subject to greater uncertainty.

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Chapter 8 Special relativity

Section 8.1 Einstein's theory of special relativity

8.1 Review

1 A. Galileo said that an object will continue in its state of motion if there is no force acting on it. If a force acts on it, its velocity will be altered.

2 C. An inertial reference frame is one in which an object remains at rest or moves with a constant linear velocity unless acted upon by a force.

3 D. They believed that all waves needed to travel in some sort of medium, so just as air is the medium for sound they proposed the existence of an aether to be the medium for light.

4 A and D. An aircraft taking off is accelerating, as is a car going around a corner, so these are non-inertial frames of reference.

5 Velocity = (1.80 – 22.25) m s^{-1} = –20.45 m s^{-1} north or 20.45 m s^{-1} south

6 A hanging pendulum in the spaceship will move from its normal vertical position when the spaceship accelerates.

7 The speed of the ball is greater for Jana than it is for Tom.

The speed of the sound is greater forwards than it is backwards for Jana, while for Tom it is the same forwards and backwards.

The speed of light is the same for Jana and Tom.

8 **a** 340 + 30 = 370 m s^{-1}

b 340 – 40 = 300 m s^{-1}

c 340 + 20 = 360 m s^{-1}

d 340 m s^{-1}

9 A. In order for the same events to be simultaneous in one inertial frame and not simultaneous in another inertial frame, time must act differently in each inertial frame of reference.

10 **a** In Anna and Ben's frame: $v = \frac{5}{0.2} = 25$ m s^{-1}, so in Chloe's frame $v = 10 - 25 = 15$ m s^{-1} backwards

b $d = vt = 15 \times 0.2 = 3$ m backwards

c 0.2 s

11 **a** $t = \frac{d}{v} = \frac{5}{50} = 0.1$ s

b 50 m s^{-1} in all frames

c $d = vt = 10 \times 0.1 = 1$ m

d 50 m s^{-1} as always

e The light had to travel ≈4 m, so $t = \frac{4}{50} = 0.08$ s (approx.)

12 Atomic clocks enable extremely short events to be timed to many decimal places. Differences in time for the same event to occur, when measured by observers in different inertial frames of reference, indicate that time is not uniform between the two inertial frames. These measurements support Einstein's special theory of relativity.

13 Muons have **very short** lives. On average, muons live for approximately 2.2 **μs**. Their speeds are measured as they travel through the atmosphere. A muon's speed is **very similar** to the speed of light. According to Newtonian laws, muons **should not** reach the Earth's surface. However, many **do**.

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Section 8.2 Time dilation

Worked example: Try yourself 8.2.1

TIME DILATION

Assume *Gedanken* conditions exist in this example. A stationary observer on Earth sees a very fast scooter passing by, travelling at $2.98 \times 10^8\,\text{m s}^{-1}$. On the wrist of the rider is a watch on which the stationary observer sees 60.0 s pass. Calculate how many seconds pass by on the stationary observer's clock during this observation. Use $c = 3.00 \times 10^8\,\text{m s}^{-1}$.	
Thinking	**Working**
Identify the variables: the time for the stationary observer is t, the proper time for the moving clock is t_0 and the velocities are v and the constant c.	$t = ?$ $t_0 = 60.0\,\text{s}$ $v = 2.98 \times 10^8\,\text{m s}^{-1}$ $c = 3.00 \times 10^8\,\text{m s}^{-1}$
Use Einstein's time dilation formula and the Lorentz factor.	$t = t_0\gamma$ $= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
Substitute the values for t_0, v, and c into the equation and calculate the answer t.	$t = \frac{60.0}{\sqrt{1 - \frac{(2.98 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$ $= \frac{60.0}{0.11528}$ $= 520\,\text{s}$

8.2 Review

1 In a device called a **light** clock, the **oscillation** of light is used as a means of measuring **time**, as the speed of light is **constant** no matter from which inertial frame of reference it is viewed.

2 'Proper time' is the time measured at rest with respect to the event. Proper times are always shorter than any other times.

3 $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.05}{\sqrt{1 - \frac{(1.75 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$= \frac{1.05}{0.81223}$$

$= 1.29\,\text{s}$

4 $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$75.0 = \frac{t_0}{\sqrt{1 - \frac{(2.30 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$t_0 = 75.0 \times 0.642$

$= 48.15\,\text{s}$

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5 $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$5.50 = \frac{t_0}{\sqrt{1 - \frac{(2.75 \times 10^8)^2}{(3 \times 10^8)^2}}}$

$t_0 = 5.50 \times 0.3996$

$= 2.20\,s$

6 $t = \frac{t_0}{\gamma} = \frac{1}{\sqrt{1 - 0.5^2}} = 1.15\,s$

7 **a** Simply the height of the clock, 1 m

b $t = \frac{d}{v} = \frac{1}{3.0 \times 10^8} = 3.33 \times 10^{-9}\,s$

c $d = vt = ct_C$

d As the distance the ship moves in Chloe's frame is $0.9ct_C$ and the height of the clock is 1 m, the distance d which the light travels is given by

$d^2 = (0.9ct_C)^2 + 1^2 = 0.81c^2t_C^2 + 1$

As this also equals $c^2t_C^2$ (from part c), we find that

$0.81c^2t_C^2 + 1 = c^2t_C^2$

$0.19c^2t_C^2 = 1$ and so

$t_C^2 = \frac{1}{0.19c^2}$, giving $t_C = 7.6 \times 10^{-9}\,s$.

e $\frac{t_C}{t_A} = \frac{7.6}{3.3} = 2.3$, which is the same as γ for $v = 90\%$ of c.

8 **a** $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$= \frac{2.2 \times 10^{-6}}{\sqrt{1 - \frac{(0.992c)^2}{c^2}}}$

$= \frac{2.2 \times 10^{-6}}{\sqrt{1 - 0.992^2}}$

$= 1.74 \times 10^{-5}\,s$ or 17.4 μs

b Non-relativistic:

$d = vt = 0.992 \times 3 \times 10^8 \times 2.2 \times 10^{-6} = 655\,m$

Relativistic:

$d = vt = 0.992 \times 3 \times 10^8 \times 1.74 \times 10^{-5} = 5178\,m$

9 $t = \frac{d}{v} = \frac{2.50 \times 10^{-2}}{2.83 \times 10^8} = 8.83 \times 10^{-11}\,s$

So the moving particle lasts for $8.83 \times 10^{-11}\,s$.

$t = t_0\gamma$

$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$8.83 \times 10^{-11} = \frac{t_0}{\sqrt{1 - \frac{(2.83 \times 10^8)^2}{c^2}}}$

$t_0 = 2.93 \times 10^{-11}\,s$

So the particle lives for $2.93 \times 10^{-11}\,s$ in the rest frame. This is reasonable, as the 'normal' lifetime should be shorter than when observed to be travelling at high speeds.

10 The equator clock is moving faster relative to the poles. It is also accelerating and hence will run slower. The effect is well below what we can detect, as the speed of the equator is 'only' about $460\,m\,s^{-1}$, which is about 1.5 millionths of c.

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Section 8.3 Length contraction

Worked example: Try yourself 8.3.1

LENGTH CONTRACTION

Assume *Gedanken* conditions exist in this example. A stationary observer on Earth sees a very fast scooter travelling by at $2.98 \times 10^8\,\text{m s}^{-1}$. The stationary observer measures the scooter's length as 45.0 cm. Calculate the proper length of the scooter, measured when the scooter is at rest. Use $c = 3.00\ 10^8\,\text{m s}^{-1}$.	
Thinking	**Working**
Identify the variables: the length measured by the stationary observer is L, the proper length for the scooter is L_0 and the velocities are v and the constant c.	$L_0 = ?$ $L = 0.450\,\text{m}$ $v = 2.98 \times 10^8\,\text{m s}^{-1}$ $c = 3.00 \times 10^8\,\text{m s}^{-1}$
Use Einstein's length contraction formula and the Lorentz factor.	$L = \frac{L_0}{\gamma}$ $= L_0\sqrt{1-\frac{v^2}{c^2}}$
Substitute the values for L, v and c into the equation and calculate the answer, L_0.	$L_0 = \frac{0.450}{\sqrt{1-\frac{(2.98\times10^8)^2}{(3.00\times10^8)^2}}}$ $= \frac{0.450}{0.11528}$ $= 3.90\,\text{m}$

Worked example: Try yourself 8.3.2

LENGTH CONTRACTION FOR DISTANCE TRAVELLED

Assume *Gedanken* conditions exist in this example. A stationary observer on Earth sees a very fast train approaching a tunnel at a speed of 0.986*c*. The stationary observer measures the tunnel's length as 123 m long. Calculate the length of the tunnel as seen by the train's driver.	
Thinking	**Working**
Identify the variables: the length seen by the driver is L, the proper length for the tunnel is L_0 and the velocity is v.	$L = ?$ $L_0 = 123\,\text{m}$ $v = 0.986c\,\text{m s}^{-1}$
Use Einstein's length contraction formula and the Lorentz factor.	$L = \frac{L_0}{\gamma}$ $= L_0\sqrt{1-\frac{v^2}{c^2}}$
Substitute the values for L_0 and v into the equation. Cancel c and calculate the answer, L.	$L = 123 \times \sqrt{1-\frac{0.986^2 \times c^2}{c^2}}$ $= 123 \times \sqrt{1-(0.986)^2}$ $= 123 \times 0.16675$ $= 20.5\,\text{m}$

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8.3 Review

1 Proper length is the length that a stationary observer measures in their own frame of reference. That is, the object (or distance) that is being measured is at rest with the observer.

2 A. Width and height are not affected as they are at right angles to the direction of motion, so a stationary observer will see a moving object with a contracted length.

3 Correct to three significant figures:

$$L = \frac{L_0}{\gamma}$$
$$= L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 1.00 \times \sqrt{1-\frac{(1.75\times10^8)^2}{(3.00\times10^8)^2}}$$
$$= 1.00 \times 0.81223$$
$$= 0.812\,\text{m}$$

4
$$L = \frac{L_0}{\gamma}$$
$$= L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 5.25 \times \sqrt{1-\frac{(2.30\times10^8)^2}{(3.00\times10^8)^2}}$$
$$= 5.25 \times 0.64205$$
$$= 3.37\,\text{m}$$

5 **a** $\gamma = \frac{3.50}{1.50} = 2.33$

thus $\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2.33} = 0.429$

$\frac{v^2}{c^2} = 1 - 0.429^2 = 1 - 0.184$

$v^2 = c^2 \times 0.816$

$v = 0.9c$ or $2.71 \times 10^8\,\text{m}\,\text{s}^{-1}$

b
$$L = \frac{L_0}{\gamma}$$
$$= L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 1.50 \times \sqrt{1-\frac{(2.71\times10^8)^2}{(3.00\times10^8)^2}}$$
$$= 1.50 \times 0.42894$$
$$= 0.643\,\text{m}$$

The fast-moving garage appears even shorter than its proper length to the car driver.

6 Proper time, t_0, because the observer can hold a stopwatch in one location and start it when the front of the carriage is in line with the watch and stop it when the back of the carriage is in line with it.

7 C. When the speed increases towards the speed of light, the distance travelled decreases.

8 **a** $\gamma = \frac{800}{400} = 2$

thus $\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2} = 0.5$

$\frac{v^2}{c^2} = 1 - 0.25$

$v^2 = c^2 \times 0.75$

$v = 0.866c$ or $2.60 \times 10^8\,\text{m}\,\text{s}^{-1}$.

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b $\gamma = 2$ so $\frac{L}{L_0} = \frac{1}{\gamma}$

$= \frac{1}{2}$

$= 0.5$

Alternatively, recognise that if the track length has been halved, then Dan appears half his thickness as well.

9 $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$= 23.5 \times \sqrt{1 - \frac{(660)^2}{(3.00 \times 10^8)^2}}$

$= 23.5 \times 1.0000$

$= 23.5\,\text{m}$

At this speed, there is no difference in length.

10 a $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$= 2.75 \times \sqrt{1 - \frac{(0.900)^2 c^2}{c^2}}$

$= 2.75 \times \sqrt{1 - (0.900)^2}$

$= 2.75 \times 0.435\,89$

$= 1.20\,\text{m}$

b The length of the fishing rod is the proper length = 2.75 m.

Section 8.4 Relativistic momentum and energy

Worked example: Try yourself 8.4.1

RELATIVISTIC MOMENTUM

a Calculate the momentum, as seen by a stationary observer, provided to an electron with a rest mass of 9.11×10^{-31} kg, as it goes from rest to a speed of 0.985*c*. Assume *Gedanken* conditions exist in this example.

Thinking	Working
Identify the variables: the rest mass is *m*, and the velocity of the electron is *v*.	$\Delta p = ?$ $m = 9.11 \times 10^{-31}\,\text{kg}$ $v = 0.985c$
Use the relativistic momentum formula.	$p = \gamma mv$
Substitute the values for *m* and *v* into the equation and calculate the answer *p*.	$p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} mv$ $= \frac{1}{\sqrt{1 - \frac{0.985^2 c^2}{c^2}}} \times 9.11 \times 10^{-31} \times 0.985 \times 3.00 \times 10^8$ $= 1.56 \times 10^{-21}\,\text{kg m s}^{-1}$

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b If three times the relativistic momentum from part (a) is applied to the electron, calculate the new final speed of the electron in terms of *c*.	
Thinking	**Working**
Identify the variables: the rest mass is *m*, and the relativistic momentum of the electron is *p*.	$p = 3 \times (1.56 \times 10^{-21})$ $= 4.68 \times 10^{-21}\,\text{kg m s}^{-1}$ $m = 9.11 \times 10^{-31}\,\text{kg}$ $v = ?$
Use the relativistic momentum formula, rearranged.	$p = \gamma mv$ $p = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}mv$ $v = \dfrac{p}{m\sqrt{\left(1 + \dfrac{p^2}{m^2c^2}\right)}}$
Substitute the values for *m* and *p* into the rearranged equation and calculate the answer *v*.	$v = \dfrac{p}{m\sqrt{\left(1 + \dfrac{p^2}{m^2c^2}\right)}}$ $= \dfrac{4.68 \times 10^{-21}}{9.11 \times 10^{-31}\sqrt{1 + \dfrac{(4.68 \times 10^{-21})^2}{(9.11 \times 10^{-31})^2(3.00 \times 10^8)^2}}}$ $= 2.995 \times 10^8\,\text{m s}^{-1}$ $= 0.998c$

Worked example: Try yourself 8.4.2

RELATIVISTIC ADDITION OF VELOCITIES

Assume *Gedanken* conditions exist in this example. An observer on Earth sees a spaceship travelling at 0.99*c* pass by. Inside the spaceship a laser beam pulse is directed towards the front of the spaceship at *c*. Calculate, in terms of *c*, the velocity of the laser beam pulse as seen by the stationary observer.	
Thinking	**Working**
Identify the variables for the velocity for the stationary observer *u*, the velocity of the ball relative to the carriage *u*′, and the velocity of the carriage *v*.	$u = ?$ $u' = 1c$ $v = 0.99c$
Use the relativistic velocity addition formula.	$u = \dfrac{u' + v}{1 + \dfrac{u'v}{c^2}}$
Substitute the values for *u*′ and *v* into the equation and calculate the answer *u*.	$u = \dfrac{(1c) + (0.99c)}{1 + \dfrac{(1c)(0.99c)}{c^2}}$ $= \dfrac{(1.99c)}{1 + \dfrac{(0.99c^2)}{c^2}}$ $= \dfrac{1.99c}{1 + 0.99}$ $= \dfrac{1.99c}{1.99}$ $= 1.00c$

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8.4 Review

1 $p = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}mv$

$= \frac{1}{\sqrt{1-\frac{(775)^2}{(3.00\times10^8)^2}}} \times 1230 \times 775$

$= 9.53 \times 10^5\,\text{kg m s}^{-1}$

2 $p = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}mv$

$= \frac{1}{\sqrt{1-\frac{(0.850)^2c^2}{c^2}}} \times 1.99264824 \times 10^{-26} \times 0.850 \times 3.00 \times 10^8$

$= 9.65 \times 10^{-18}\,\text{kg m s}^{-1}$

3 Since $v = c$, $p = mv$

$= 1.99264824 \times 10^{-26} \times 800$

$= 1.59 \times 10^{-23}\,\text{kg m s}^{-1}$

4 $E_k = (\gamma - 1)mc^2$

$= \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)mc^2$

$= \left(\frac{1}{\sqrt{1-\frac{0.750^2c^2}{c^2}}} - 1\right)(0.0123)(3.00 \times 10^8)^2$

$= \left(\frac{1}{\sqrt{1-0.750^2}} - 1\right)(0.0123)(3.00 \times 10^8)^2$

$= (1.5118 - 1)(0.0123)(3.00 \times 10^8)^2$

$= (0.5118)(0.0123)(3.00 \times 10^8)^2$

$E_k = 5.67 \times 10^{14}\,\text{J}$

5 $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}(0.0123)(0.750 \times 3.00 \times 10^8)^2$

$= 3.11 \times 10^{14}\,\text{J}$

6 B. Relativistic kinetic energy depends on the momentum of the arrow. For the very fast arrow, the relativistic momentum is larger than the classical momentum.

7 $u' = \frac{u - v}{1 - \frac{uv}{c^2}}$

$= \frac{(0.95c) - (0.98c)}{1 - \frac{(0.95c)(0.98c)}{c^2}}$

$= \frac{(-0.03c)}{1 - (0.931)}$

$= \frac{(-0.03c)}{(0.069)}$

$= -0.435c$

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CHAPTER 8 REVIEW

1 No object can travel at or beyond the speed of light, so the value of $\frac{v^2}{c^2}$ will always be less than 1.

The number under the square root sign will also, therefore, be a positive number less than 1.
The square root of a positive number less than 1 will always be less than 1 as well.

Note, however, that when v is very small $\frac{v^2}{c^2}$ is also very small and so the number under the square root sign will be very close to 1. The result is a number very close to 1. Some calculators may not be able to distinguish a number so close to 1, but this is just due to the limitations of the calculator.

2 The speed is 0.000 167 of c and so $\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(0.000167c)^2}{2c^2} = 1.000\,000\,014$

3 A (postulate 2) and C (postulate 1)

4 At the poles. The Earth has a very small circular acceleration, which is negligible for most purposes; however, at the poles it is even less.

5 C. There is no 'fixed space' in which to measure absolute velocities; you can only measure them relative to some other frame of reference.

6 Space and time are interdependent—motion in space reduces motion in time.

7 All observers will see the light travel at $3 \times 10^8\,\mathrm{m\,s^{-1}}$. According to Einstein's second postulate, the speed of light will always be the same no matter what the motion of the light source or the observer.

8 A and B. We are in the same frame in either case. C and D may be true, but they are not sufficient conditions as we must be in the same frame. (C did not specify with respect to what we were stationary.)

9 B. Crews A and B will see each other normally as there is no relative velocity between them. They will both see C and the Earthlings moving in slow motion, as the Earth has a high relative velocity.

10 You could not tell the difference between (i) and (iii), but in (ii) you could see whether an object such as a pendulum hangs straight down.

11 In your frame of reference time proceeds normally. Your heart rate would appear normal. As Mars is moving at a high speed relative to you, people on Mars appear to be in slow motion as time for them, as seen by you, will be dilated.

12 $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{20.0}{\sqrt{1 - \frac{(2.00 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$= 26.8\,\mathrm{s}$$

13 **a** $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$1.5 = \frac{t_0}{\sqrt{1 - \frac{(2.25 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$t_0 = 1.5 \times 0.6614$$
$$= 0.992\,\mathrm{s}$$

b 0.992 s (the swimmer sees the pool clock as t_0)

14 C. The remaining twin ages faster *during both the acceleration and deceleration phases* because the twin who travels experiences non-inertial frames of reference when accelerating and decelerating. This is when the travelling twin sees the twin at home ageing rapidly.

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15 a $L = \frac{L_0}{\gamma}$ and $\frac{L}{L_0} = \frac{1}{2}$

Thus $\sqrt{1 - \frac{v^2}{c^2}} = 0.5$

$\frac{v^2}{c^2} = 1 - 0.25$

$v^2 = c^2 \times 0.75$

$v = 0.866c$ or $2.598 \times 10^8\,\text{m s}^{-1}$

b No, it can't have doubled to over c! The contraction has doubled so this time $\gamma = 4$.

Then $\sqrt{1 - \frac{v^2}{c^2}} = 0.25$

$\frac{v^2}{c^2} = 1 - 0.0625$

$v^2 = c^2 \times 0.9375$

$v = 0.968c$ or $2.90 \times 10^8\,\text{m s}^{-1}$

16 a $t = t_0\gamma$

$= \frac{1.00}{\sqrt{1 - \frac{v^2}{c^2}}}$

$= \frac{1.00}{\sqrt{1 - \frac{(2.4 \times 10^8)^2}{(3 \times 10^8)^2}}}$

$= 1.67\,\text{s}$

b Length:

$L = \frac{L_0}{\gamma}$

from part (a), $\gamma = 1.67$

$L = \frac{3.00}{1.67}$

$= 1.80\,\text{m}$

The height is unchanged at 1.0 m.

17 a $t = \frac{d}{v} = \frac{5}{0.9} = 5.6$ years

b $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = 2.29$

$t_0 = \frac{t}{\gamma}$

$= \frac{5.6}{2.29}$

$= 2.45$ years

c Raqu sees the distance as only

$L = \frac{L_0}{\gamma} = \frac{5}{2.29} = 2.183$ light-years

18 a At $8000\,\text{m s}^{-1}$, $\frac{v}{c} = 2.7 \times 10^{-5}$ and γ will have a value of

$\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(2.7 \times 10^{-5})^2}{2} = 1 + 3.6 \times 10^{-10}$

The difference (in mm) will therefore be $4 \times 10^9 \times 3.6 \times 10^{-10} = 1.4$ mm—hardly a problem!

b No, the motion is perpendicular to the north–south direction, so this dimension is not affected.

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19 a $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.995c)^2}{c^2}}} = \frac{1}{\sqrt{1-(0.995)^2}} = 10.01$

b No, they don't experience any difference in their own time frame.

c $t = \frac{25}{0.995} = 25.1$ years

About 25.1 years from our frame of reference.

d 2.51 years as $\gamma = 10$

e No! They see the distance between Earth and Vega foreshortened because of the high relative speed, so to them the distance is only about 2.5 light-years.

20 Earth observer: the observer will not measure the proper time of the muon's life span. Instead they will see that the muon's time is slow according to the equation $t = t_0\gamma$ where t_0 is the rest lifespan of the muon. The result is that the observer sees the muon live a much longer time, *t*, and therefore make it to the Earth's surface.

Muon: the muon will see the Earth approach at a very high speed (approx. 0.992*c*) and will see the distance contracted. It will not be 15 km, but instead be much shorter according to the equation $L = \frac{L_0}{\gamma}$. The distance the muon travels is *L*.

21 $u' = \frac{(u-v)}{1-\frac{uv}{c^2}}$

$= \frac{(0.85-0.58)c}{1-\frac{(0.85)(0.58)c^2}{c^2}}$

$= \frac{0.27c}{1-0.493} = 0.533c$

22 a $p = mv = 950 \times 0.65 \times 3 \times 10^8 = 1.85 \times 10^{11}\,\text{kg m s}^{-1}$

b $p_v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.85 \times 10^{11}}{\sqrt{1-0.65^2}}$

$= 2.44 \times 10^{11}\,\text{kg m s}^{-1}$

c As the velocity of the starship approaches *c*, the Lorentz factor becomes significantly larger.

23 a $u = \frac{u'+v}{1+\frac{u'v}{c^2}} = \frac{0.765c + 0.67c}{1+\frac{0.765c \times 0.67c}{c^2}} = 0.949c$

b Different velocities means they must be in different reference frames regardless of the direction in which each is moving.

24 a $p_v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.90 \times 0.85 \times 3 \times 10^8}{\sqrt{1-0.85^2}}$

$= 9.20 \times 10^8\,\text{kg m s}^{-1}$

b $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.90 \times 9 \times 10^{16}}{\sqrt{1-0.85^2}}$

$= 3.25 \times 10^{17}\,\text{J}$

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Chapter 9 The Standard Model

Section 9.1 Particles of the Standard Model

9.1 Review

1 The particles of the Standard Model have been classified into three main groups.
- Force-carrier particles such as the gluon are called **gauge bosons**.
- Fundamental particles, such as the electron, that can be found individually and do not experience the strong nuclear force are called **leptons**.
- The particles that are largest in number, and are made of quarks, are called **hadrons**.

2 The Standard Model of particle physics describes three of the fundamental forces in the universe but does not incorporate the *gravitational force*. It predicts that all matter is made of *quarks* and *leptons*. The main difference between these two groups of particles is that quarks experience the *strong nuclear force* but *leptons* do not. Forces such as the *strong nuclear force* are mediated by *force-carrier particles* called *gauge bosons* that are different for each force. *Gluons* are the *force-carrying particles* for the *strong nuclear force*, *photons* carry the *electromagnetic force* and *Z, W^- and W^+ particles* carry the *weak nuclear force*.

3 A. All atoms contain protons and electrons. Electrons are leptons and protons are made of quarks. Electrons are held around the nucleus by the electromagnetic force, which requires a gauge boson (the photon), and quarks are held together by the gauge boson for the strong nuclear force, a gluon.

4 The first force, the weak nuclear force, is involved in the decay of the neutron.

The second force is responsible for electrostatic attraction. The proton's attraction to the electron is via the electromagnetic force.

The third force attracts the hydrogen atom towards Earth. When the atom drifts towards the base of the container it is being attracted by the force of gravity.

The one force that is not involved is the strong nuclear force.

5 B. Quarks are always bound in groups to form mesons or baryons. A is incorrect: quarks carry charge and so do some leptons (e.g. electrons). C is incorrect: lepton number is conserved but there is not a specific 'quark number'. D is incorrect. If quarks and leptons both experience the strong nuclear force, they would both be able to produce composite particles.

6 **a** gluon: gauge boson
b neutrino: lepton
c neutron: hadron
d photon: gauge boson
e electron: lepton
f muon: lepton
g proton: hadron
h tau: lepton

7 B. The Standard Model says that forces between two particles are mediated by the exchange of force-carrier particles. In the analogy for the repulsive force, throwing and catching a ball causes an exchange of momentum, and the skaters move apart.

8

	Fermions		
Type of fermion	quark		lepton
Gauge boson	gluon		photon
Charge	$+\frac{2}{3}, -\frac{1}{3}$		0, –1
Hadrons formed	baryon	meson	
Number of particles in the hadron	3	2	

9 D. The up quark has a charge of $+\frac{2}{3}$ and the down quark a charge of $-\frac{1}{3}$. The proton must be made of an up, an up and a down to give a net charge of +1. A neutron must be made of an up, a down and a down to give a net charge of 0.

10 **a** e^+ **b** $\bar{\nu}_\tau$ **c** ν_τ **d** μ^- **e** μ^+

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Section 9.2 Interactions between particles

Worked example: Try yourself 9.2.1

ELECTRON-VOLTS AND JOULES

Calculate the energy given to an electron accelerated across a potential difference of 2.5×10^6 V. Give your answer in joules correct to two significant figures.	
Thinking	**Working**
Determine the number of electron-volts of energy that the particle gains.	$1\,\text{eV} \times 2.5 \times 10^6 = 2.5 \times 10^6\,\text{eV}$ or 2.5 MeV
Convert the value in eV to J.	$2.5 \times 10^6 \times 1.60 \times 1^{-19}\,\text{J} = 4.0 \times 10^{-13}\,\text{J}$

Worked example: Try yourself 9.2.2

CONSERVATION OF MASS–ENERGY

A 1.20 MeV gamma-ray photon interacts with an atomic nucleus and an electron–positron pair is produced. Each particle created has a mass of 0.511 MeV. How much kinetic energy is carried away by the particle–antiparticle pair produced?	
Thinking	**Working**
The energy of the gamma-ray photon is conserved and is equal to the total mass–energy of the particles produced plus the kinetic energy (E_k) of those particles.	$1.20\,\text{MeV} = 2 \times 0.511\,\text{MeV} + E_k$
Solve for the kinetic energy.	$1.20 - 1.022 = E_k$
Calculate the answer.	$E_k = 0.178\,\text{MeV}$

Worked example: Try yourself 9.2.3

CONSERVATION OF CHARGE

A physicist proposed that the products of an observed collision of a proton and a neutron were two protons, a neutron and an antiproton. Determine whether this reaction is allowed according to the law of conservation of charge.	
Thinking	**Working**
To find out if this reaction is allowed, calculate the total charge present before and after the interaction.	Proton's charge = +1 Neutron's charge = 0 Antiproton's charge = −1 Total charge before = (+1) + 0 = +1 Total charge after = 2 × (+1) + 0 + (−1) = +1
Compare the total charge before and after the interaction. If they are equal then the reaction is allowed; if they are different then the reaction is forbidden.	The total charge before and after is the same. Therefore, this proposed reaction is allowed according to the law of conservation of charge.

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Worked example: Try yourself 9.2.4

CONSERVATION OF BARYON NUMBER

A physicist proposed that the products of an observed collision of a proton and a neutron were two protons, a neutron and an antiproton. Is this reaction allowed according to the law of conservation of baryon number?	
Thinking	**Working**
The law of conservation of baryon number states that the net number of baryons remains constant in any process. To find out if this reaction is allowed, calculate the total baryon number before and after the interaction.	Proton $B = +1$ Neutron $B = +1$ Antiproton $B = -1$ Total B before = (+1) + (+1) = +2 Total B after = 2 × (+1) + (+1) + (−1) = +2
Compare the total baryon number before and after the interaction. If they are equal then the reaction is allowed according to the law of conservation of baryon number. If they are different then the reaction is forbidden, as baryon number is not conserved.	The total baryon number before and after the collision is the same. Therefore, this proposed reaction is allowed according to the law of conservation of baryon number.

Worked example: Try yourself 9.2.5

CONSERVATION OF LEPTON NUMBER

A physicist proposed that the observed decay of a meson (made of two quarks) called a pion (π^-) produced an electron neutrino (ν_e), a muon neutrino (ν_μ) and an antimuon (μ^+). Is this reaction allowed according to the law of conservation of lepton number?	
Thinking	**Working**
Identify which types of leptons are involved in the interaction. These need to be considered separately.	Electron/electron neutrino, L_e: yes Muon/muon neutrino, L_μ: yes Tau/tau neutrino, L_τ: no
To find out if this reaction is allowed, calculate the total muon lepton number L_μ before and after the interaction.	Muon neutrino $L_\mu = +1$ Antimuon $L_\mu = -1$ Total before $L_\mu = 0$ Total after $L_\mu = 0$
Compare the total muon lepton number before and after the interaction. If they are equal then the reaction is allowed. If they are different then the reaction is forbidden.	The total muon lepton number before the reaction is the same as the total muon lepton number after the reaction. Therefore, this proposed reaction is allowed according to the law of conservation of muon lepton number.
To find out if this reaction is allowed, calculate the total electron lepton number L_e before and after the interaction.	Electron neutrino $L_e = +1$ Total before $L_e = 0$ Total after $L_e = +1$
Compare the total electron lepton number before and after the interaction. If they are equal then the reaction is allowed. If they are different then the reaction is forbidden.	The total electron lepton number before the reaction is different from the total electron lepton number after the reaction. Therefore, this proposed reaction is forbidden according to the law of conservation of electron lepton number.

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9.2 Review

1 Use the time axis to indicate the sequence of events in this interaction and remember that the direction of the arrows does not indicate the direction of travel of particles.

The photon that is exchanged tells you which force is involved and the symbols of the particles involved indicate their identity.

This interaction results in a force of repulsion between two positrons. This interaction involves the electromagnetic force as it is mediated by the exchange of a photon.

2 Use the time axis to indicate the sequence of events in this interaction.

The gauge boson involved tells you which force is involved and the symbols of the particles involved indicate their identity.

This interaction is the decay of a muon, which involves the weak nuclear force. It results in the production of a muon neutrino (ν_μ), an antielectron neutrino ($\bar{\nu}_e$) and an electron (e^-).

3 An electron gains 1 eV of energy for every volt it is accelerated through.

At half way, the electron has passed through half of the potential difference.

$1\text{ eV} \times 0.5 \times 1.4 \times 10^3 = 0.70 \times 10^3\text{ eV}$

4 According to the law of conservation of charge the decay is forbidden as the initial overall charge is 0 and the final overall charge is +1. Therefore, charge is not conserved.

According to the law of conservation of energy the decay is forbidden as the initial energy in the neutron is 939.57 MeV and the total energy of particles after the interaction is greater than the total initial energy. Therefore, energy is not conserved.

5 Annihilation involves the destruction of a particle and its antiparticle, and pair production involves the production of a particle and its antiparticle.

Annihilation involves the conversion of mass into energy, and pair production involves the conversion of energy into mass.

When an electron meets a positron, annihilation occurs, converting the masses of the electron and positron into the energy of two photons.

6 Conservation laws require a quantity or quantum number after an interaction or decay to be the same as the quantity or quantum number's initial value. These laws can be used to predict if a reaction is allowed or forbidden. Physicists can then observe interactions or decays to test their predictions.

7 C. The vertical axis represents time increasing vertically. Products of a reaction remain at the end of the event. It shows a decay in which a proton is converted into a neutron; the other products are a positron and an electron neutrino.

8 Charge conservation requires the total charge before the decay to equal the total charge after the reaction.

Conservation of baryon number occurs when the baryon number remains constant. Conservation of lepton number occurs when the electron lepton number, the muon lepton number and the tau lepton number all remain constant.

a lepton number (electron lepton number and muon lepton number are not conserved)

b baryon number

c charge

9 The law of conservation of electron lepton number leads to the conclusion that the missing product is an antielectron neutrino ($\bar{\nu}_e$). The reaction does not violate charge conservation or baryon conservation.

10 The third decay product must be an antielectron neutrino ($\bar{\nu}_e$) in order to obey the law of conservation of electron lepton number.

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Section 9.3 Particle accelerators

Worked example: Try yourself 9.3.1

ENERGIES OF PARTICLES

A proton with a rest mass of 1.672×10^{-27} kg is accelerated to 0.95*c*. Calculate the total energy of the particle in J and MeV. Give your answer to two significant figures.

Thinking	Working
Identify the equation you need to use, based on the information you know already and the unknown you need to calculate.	The equation that is needed: $E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$
Identify relevant known data.	$m_0 = 1.672 \times 10^{-27}$ kg $v = 0.95c$ $c = 3.0 \times 10^8$ m s^{-1} $E = ?$
Convert any values to SI units if needed and substitute into the equation.	$E = \frac{1.672 \times 10^{-27} \times (3.0 \times 10^8)^2}{\sqrt{1-\frac{0.95^2c^2}{c^2}}}$ $= 4.8 \times 10^{-10}$ J
Convert J to MeV.	1 eV = 1.6×10^{-19} J 1 MeV = 1.6×10^{-13} J $\therefore E$ (MeV) $= \frac{4.8 \times 10^{-10}}{1.6 \times 10^{-13}}$ $= 3.0 \times 10^3$ MeV
Check your answer. This amount of energy should be greater than the rest energy of the proton.	$E = m_0c^2$ $= 1.672 \times 10^{-27} \times (3.0 \times 10^8)^2$ $= 1.5 \times 10^{-10}$ J 1 eV = 1.6×10^{-19} J 1 MeV = 1.6×10^{-13} J $\therefore E$ (MeV) $= \frac{1.5 \times 10^{-10}}{1.6 \times 10^{-13}}$ = 940 MeV The answer is 3.2 times this value, so significant dilation of mass has occurred at 0.95*c*. The answer seems reasonable.

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Worked example: Try yourself 9.3.2

BEAM BENDING

A proton travels at $0.9 \times 10^8\,m\,s^{-1}$ in a direction perpendicular to a uniform magnetic field of strength 0.7 T. The rest mass of a proton is $m_0 = 1.672 \times 10^{-27}$ kg and the charge on the proton is $+1.6 \times 10^{-19}$ C. Assume that the increase in mass of the particle at this velocity is negligible. Calculate the radius of the circular path of the proton. Give your answer to one significant figure.	
Thinking	**Working**
Rearrange the equation that represents the relationship between the Lorentz force and the centripetal force to make *r* the subject.	$F = qvB = \frac{mv^2}{r}$ $r = \frac{mv}{qB}$
Identify relevant known data.	$m_0 = 1.672 \times 10^{-27}$ kg $v = 0.9 \times 10^8\,m\,s^{-1}$ $B = 0.7$ T $q = 1.6 \times 10^{-19}$ C
Substitute the values into the equation.	$r = \frac{1.672 \times 10^{-27} \times 0.9 \times 10^8}{1.6 \times 10^{-19} \times 0.7}$ $= 1.34$ m $= 1$ m

9.3 Review

1 B. The wavelength of a particle influences the detail that it can resolve in a collision. Increasing the momentum decreases the wavelength. The shorter the wavelength of an accelerated particle, the finer the detail it can provide from collisions. Increasing mass or velocity increases momentum.

2 **a** Bending magnets apply a centripetal force to particles.
b Beam pipe allows an evacuated path for particles to be produced.
c Accelerating cavities increase the velocity of the particles.
d Detectors identify and quantify collision products.

3 $E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$m_0 = \frac{E}{c^2}\sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{3.78 \times 10^{-10}}{c^2}\sqrt{1 - \frac{(0.999c)^2}{c^2}}$$

$$= 1.88 \times 10^{-28}\,kg$$

4 $F = \frac{mv^2}{r} = Bqv$

$$r = \frac{mv}{Bq}$$

$$= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \times \frac{v}{Bq}$$

$$= \frac{1.672 \times 10^{-27} \times 0.999c}{\sqrt{1 - \frac{0.999^2c^2}{c^2}} \times 7.0 \times 1.6 \times 10^{-19}}$$

$$= 10.0\,m$$

$r = 10$ m (1 significant figure)

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5 B. The gauge boson for the electromagnetic force is the photon and for the weak nuclear force it is the W and Z bosons. The W and Z bosons have high mass and the photons have no mass. Although the weak nuclear and electromagnetic forces are unified in the Standard Model and in the early universe, this does not explain the differences.

6 The development that enabled the discovery of the W and Z bosons was the development of higher-energy accelerators in which particles were able to be collided with sufficient energy to produce heavy bosons. This was done by colliding protons and antiprotons in the Super Proton Synchrotron at CERN.

7 D. All conservation laws listed as well as others not listed.

8 The right-hand palm rule indicates that the particle is experiencing a force in the direction experienced by a negative particle.

$$F = \frac{mv^2}{r} = Bqv$$

$$m = \frac{Bqr}{v}$$

$$= \frac{(5.0)(1.6 \times 10^{-19})(10)}{0.4c}$$

$$= 6.7 \times 10^{-26}\,\text{kg}$$

9 $$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{1.672 \times 10^{-27} \times c^2}{\sqrt{1 - \frac{0.99^2c^2}{c^2}}}$$

$$= 1.06 \times 10^{-9}\,\text{J}$$

Double this energy $= 2.12 \times 10^{-9}$ J

$$E = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0c^2}{E}$$

$$= \frac{1.672 \times 10^{-27} \times c^2}{2.12 \times 10^{-9}}$$

$$= 0.070981$$

$v = 0.9975c$

$\Delta v = 0.9975c - 0.99c$

$= 0.0075c$

$$\frac{\Delta v}{v} = \frac{0.0075}{0.99}$$

$= 0.0076$

Velocity would be 1.0076 times original velocity.

10 $$F = Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

if $v \times 2$, then $r \times 2$

The radius doubles.

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Section 9.4 Expansion of the universe

9.4 Review

1 A. This is consistent with the observation that light is moved towards the red end of the spectrum (redshifted) for an object moving away from an observer. Wavelength is longer and frequency is lower as light shifts towards red.

2 Order: galaxy 2, galaxy 1, galaxy 3

Hubble's law shows that galaxies at greater distances have greater recessional velocities. Red shift is proportional to recessional velocities. Galaxy 3 has the greatest red shift and, therefore, the greatest distance.

3 The Hubble constant represents the rate of expansion of the universe today, i.e. how fast galaxies are receding from us, with each unit of distance they are away from us. For each Mpc a galaxy is away from us, it is receding from us 67.80 $km\,s^{-1}$ faster.

4 $v = H_0 d$

Convert $m\,s^{-1}$ to $km\,s^{-1}$

$4 \times 10^7\,m\,s^{-1} = 4 \times 10^4\,km\,s^{-1}$

$$d = \frac{v}{H_0}$$
$$= \frac{4 \times 10^4}{72}$$
$$= 6 \times 10^2\,Mpc$$

5 In a conventional explosion, all matter moves outwards from a single point. However, space expands uniformly in whatever direction you look and from wherever you look. There is no 'centre' of the three-dimensional space of the universe.

6 A, C and D. All the stars and galaxies appear to be receding, the stars and galaxies within space are not expanding and the same recession of stars and galaxies is seen throughout space.

B is not a correct answer, as there is no centre to the expansion and therefore the universe isn't expanding relative to Earth.

7 The cosmic microwave background radiation was emitted when neutral atoms formed in the early universe. The early radiation continues to exist in the universe but has expanded with the universe.

8 Shortly after the Big Bang, the universe was full of very high temperature radiation. This radiation still fills the universe, but the expansion has cooled it to less than 3 K. The Steady State theory has no explanation for the presence of such radiation. Instead, the Steady State theory says the universe has been the same forever.

9 gravity, strong nuclear, weak nuclear, electromagnetic

The force that is the most difficult to unify with those in the Standard Model of particle physics separated from the others first (gravity). The force that holds hadrons together separated next (strong force).

10 Both theories account for the large number of hydrogen and helium atoms in the universe, but in different ways. The Steady State theory says that hydrogen atoms are created at a rate of one atom per cubic metre per 300 000 years, while Big Bang theory says that all of the hydrogen and helium was created at the Big Bang out of nothing.

CHAPTER 9 REVIEW

1 All quarks experience the strong nuclear force; leptons do not. Quarks also have non-integer (fractional) charges; leptons have charges of –1 or 0. Leptons are generally not found in the nucleus, while quarks are found in the nucleus. Leptons can exist as free particles but quarks cannot.

2 C. Since protons and neutrons are made of quarks, all protons and neutrons are attracted to adjacent protons and neutrons within the nucleus.

3 **a** photon

b gluon

c W^+, W^- and Z

4 The Standard Model is based on the assumption that forces arise through the exchange of particles called gauge bosons (or just bosons). Each of the three forces is mediated by a different particle. The strong nuclear force is mediated by the gluon, the electromagnetic force by the photon and the weak nuclear force by W^+, W^- and Z bosons.

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5 The particles of the Standard Model include gauge bosons, leptons and hadrons. Both gauge bosons and leptons are fundamental particles, which means that they are not made up of other particles, while hadrons are made of two or three smaller particles called quarks. Gauge bosons are force-carrying particles, while leptons and hadrons are not.

6 As yet the force of gravity is not considered to be part of the Standard Model.

7 $W = qV = 1.60 \times 10^{-19} \times 1 \times 10^{10} = 1.6 \times 10^{-9}$ J

8 The law of conservation of baryon number and the law of conservation of muon lepton number are violated by this reaction.

$\pi^+ \rightarrow n + \mu^+$

Charge: (+1) = (0) + (+1) is conserved

Baryon number: (0) ≠ (+1) + (0) is not conserved

Lepton number: (0) ≠ (0) + (–1) is not conserved

9 Colliding two particles travelling in opposite directions is possible in a synchrotron, but not in a linear accelerator. In a synchrotron, the sum of the momentums before the collision will be close to zero and so the sum of the momentums carried by particles produced in the collisions is relatively small. This results in most of the energy of the collision being available to create the mass of new particles.

10 $F = Bqv = \frac{mv^2}{r}$

$$B = \frac{mv}{rq}$$

$$= \frac{(1.672 \times 10^{-27})(1.2 \times 10^8)}{(2)(1.6 \times 10^{-19})}$$

$$= 0.6\text{ T}$$

11 $E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}}$

$$\frac{E}{E_{rest}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}}$$

$$= 5$$

Increase = 4

% increase = 400%

12 Rest mass and relativistic kinetic energy. At relativistic speeds, classical equations no longer apply. The total energy of the particle is its rest mass plus the energy it has due to its motion. This energy due to motion can be described as relativistic kinetic energy, and can be quantified by calculating a dilated or relativistic mass.

13 Within a linear accelerator, an electric field does work on charged particles by alternating the potential of the drift tubes along the accelerator at a constant rate. This causes particles to gain kinetic energy.

14 It had been found that the Cepheids varied in brightness and that the period of this variation was related to the intrinsic brightness. By comparing intrinsic and apparent brightness, the distance could be found.

15 This reaction involved baryons, some of which are charged, so conservation of baryon number and conservation of charge are relevant here. According to the conservation of baryon number the reaction is allowed. The baryon number is 2 before (proton and neutron) and 2 after the reaction (two protons). The law of conservation of charge is also satisfied. The total charge is +1 before (proton) and +1 after this reaction (two protons and a π^-). Therefore, by both these laws this reaction is allowed.

16 Annihilation describes the process by which a particle and its corresponding antiparticle collide and are converted into photons. The opposite process is pair production, in which an elementary particle and its antiparticle are created from photons.

17 $E_{ph} = 2(E_{mass} + E_k)$

$$= 2(0.511 + 0.250)$$

$$= 2(0.761)$$

$$= 1.52\text{ MeV}$$

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18 $70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} = \frac{70}{3.09 \times 10^{19}}\,\text{s}^{-1} = 2.27 \times 10^{-18}\,\text{s}^{-1}$

$\frac{1}{H_0} = \frac{1}{2.27 \times 10^{-18}}$

$= 4.41 \times 10^{17}\,\text{s}$

Age of universe $\frac{1}{H_0} = 1.4 \times 10^{10}$ years

19 Edwin Hubble plotted a scatter graph of recession velocity (determined from the amount of redshift of a galaxy's spectrum) against distance to the galaxy (determined using Cepheid variables). This graph showed a straight-line relationship, indicating that recession velocity is proportional to the distance of a galaxy.

If distant galaxies are all observed to be moving away from us, you might first assume that the galaxies are all moving apart and you are at the centre of the universe. If we were to run time backwards, this would result in all the galaxies having come from the same region of space and therefore the universe must be expanding.

Note: Under the assumption that there is nothing remarkable about our location in the universe, we must conclude that the galaxies are not all flying away from us and that it is in fact the space between the galaxies that is getting bigger.

20 A. The convention for the direction of an electric field is the direction in which a positive charge experiences a force.

Work is done by the field when the field exerts a force on a charge and it moves from a higher potential to a lower one.

The field does work on the electrons in the opposite direction to the field. Therefore, the field needs to remain in the opposite direction to the motion of the electron along the accelerator to maintain a force on the electron, causing acceleration. Linear accelerators operate at one frequency for a certain particle and have drift tubes of increasing length.

21 B. The prediction of the W and Z bosons was part of electroweak theory and the assumption of the existence of gauge bosons for each fundamental force. An experiment cannot prove a theory to be correct or wrong. It can only support or refute a hypothesis. The hopes of winning the Nobel prize would not be justification to conduct these experiments. The Standard Model contains several components. It is founded on the theory of special relativity and quantum mechanics.

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Chapter 10 Practical investigation

Section 10.1 Designing and planning the investigation

10.1 Review

1 **a** If the voltage is measured in units of number of batteries then it is a discrete value.
b If the voltage is measured with a voltmeter then the voltage would be continuous.

2 qualitative

3 A. Hypothesis 1 is the best answer as it is a definite statement of the relationship between the independent and dependent variables.

4 **a** valid
b reliable
c accurate

5 **a** the tension in the elastic band
b the initial launch velocity of the elastic band
c the same elastic band, elastic band held in the same way, elastic band launched in the same direction, elastic band placed on the finger in the same way

Section 10.2 Conducting investigations and recording and presenting data

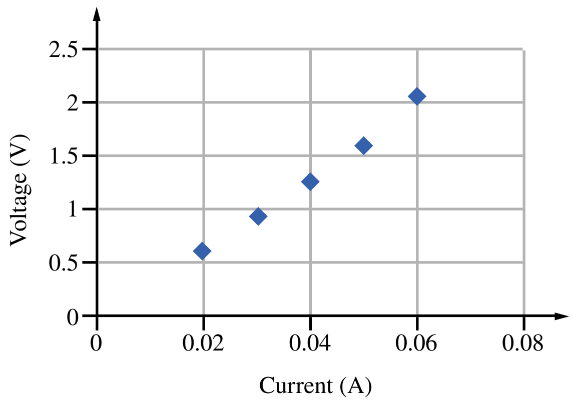
10.2 Review

1 **a** systematic error
b random error

2 Give answer to two significant figures, as this is the smallest number of significant figures in the data provided.

3 **a** mean = (21 + 28 + 19 + 19 + 25 + 24) ÷ 6 = 22.7
b mode = 19
c median = 22.5
d uncertainty in the mean: 28 – 23 = ±5

4



5 The trend can be represented as a line of best fit on the graph.

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Section 10.3 Discussing investigations and drawing evidence-based conclusions

10.3 Review

1 A linear graph shows the proportional relationship between two variables.
2 an inversely proportional relationship
3 a directly proportional relationship
4 time restraints and limited resources
5 An increase in current from 0.03 A to 0.05 A produced an increase of 0.88 V across the resistor.

CHAPTER 10 REVIEW

1 A hypothesis is a prediction, based on evidence and prior knowledge, to answer the research question. A hypothesis often takes the form of a proposed relationship between two or more variables.
2 Dependent variable: flight displacement
Independent variable: release angle
Controlled variable: (any of) release velocity, release height, landing height, air resistance (including wind)
3 **a** the acceleration of the object
b the vertical acceleration of the falling object
c the rate of rotation of the springboard diver
4 Elimination, substitution, isolation, engineering controls, administrative controls, personal protective equipment
5 $6.8 \pm 0.4\,\text{cm}\,\text{s}^{-1}$
6 the mean
7 an exponential relationship
8 This graph should show a straight line with a positive gradient.
9 Any issues that could have affected the validity, accuracy, precision or reliability of the data plus any sources of error or uncertainty.
10 Bias is a form of systematic error resulting from a researcher’s personal preferences or motivations.

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Pearson

Unit 4 Review

Section 1: Short response

1 An electromagnetic wave is a propagating wave consisting of oscillations of electric and magnetic fields—this is a more formal definition of what light is. The relationship between energy, wavelength and frequency can be summarised in the wave equation $E = hf = \frac{hc}{\lambda}$: the higher the energy, the shorter the wavelength and the higher the frequency.

Electromagnetic waves are different from mechanical waves in that they do not require a medium in which to propagate.

2 **a** The photoelectric effect demonstrates light exhibiting particle-like behaviour. When a beam of light of high enough energy is incident on a metal, electrons in the metal will be ejected. These electrons can then be collected at the collector electrode and a photocurrent detected. If a negative potential is applied up until the point at which current is reduced to zero, the exact kinetic energy of the electrons can be determined. Several observations were made from this:

- When light intensity increases, the photocurrent increases.
- There is a negative voltage for which no photoelectrons reach the collector. This is known as the stopping voltage. For a particular metal, each frequency of light will give a characteristic stopping voltage. This value is independent of light intensity.

The characteristics of the photoelectric effect could not be explained using a wave model of light. According to the wave model, the frequency of the light should be irrelevant to whether or not photoelectrons are ejected. Since a wave is a form of continuous energy transfer, it would be expected that even low-frequency light should transfer enough energy to emit photoelectrons if left incident on the metal for long enough. Similarly, the wave model predicts that there should be a time delay between the light striking the metal and photoelectrons being emitted, as the energy from the wave builds up in the metal over time.

b When given enough momentum and fired through small slits (or more complex objects, such as a crystalline structure) electrons diffract and produce interference patterns with one another. Furthermore, it can be shown that electrons with the same wavelength as a particular frequency of light will produce the same diffraction pattern. This wavelength is predicted by de Broglie's formula for matter waves: $\lambda = \frac{h}{p}$.

3 There are two equations that can be used to solve the problem:

$E = hf$ and $f = \frac{c}{\lambda}$

For the first packet of light:

$f_1 = \frac{c}{220 \times 10^{-9}}$

For the second packet of light:

$11.24 = 4.14 \times 10^{-15} \times f_2$

Since the second beam of light is twice as energetic as the first:

$5.62 = 4.14 \times 10^{-15} \times f_1$

$\therefore f_1 = \frac{5.62}{4.14 \times 10^{-15}} = 1.36 \times 10^{15}$ Hz

$\therefore c = f_1\lambda_1 = 1.36 \times 10^{15} \times 220 \times 10^{-9}$

$c = 2.99 \times 10^{8}\,\text{m s}^{-1}$

4
- Only certain frequencies of light will eject photoelectrons.
- There is no time difference between the ejection of photoelectrons by light of different intensities.
- The maximum kinetic energy of the ejected photoelectrons is the same for different light intensities of the same frequency.

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5 **a** 1 The laws of physics are the same in all inertial frames of reference.

2 The speed of light is a constant for all observers.

b First, identify the variables for the velocity of the stationary observer, *u*, the velocity of the faster ship in the slower ship's frame of reference, and the velocity of the moving frame of the slower ship, *v*.

Using the formula for relativistic addition of velocities:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$= \frac{0.950c - 0.800c}{1 - \frac{(0.950c)(0.800c)}{c^2}}$$

$$= \frac{0.150c}{1 - 0.760}$$

$u' = 0.625c$

6 Moving objects contract, and so the 1.50 m ruler should be shorter in the moving frame. First find the gamma factor and then use the length contraction formula:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(1.35 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$$

$\gamma = 1.12$

$$L' = \frac{L}{\gamma}$$

$$= \frac{1.50}{1.12}$$

$L' = 1.34\,\text{m}$

7 Weak nuclear for the decay, electromagnetic attracts the proton and the electron together and gravity accelerates the atom to the base of the container; the strong nuclear is not involved.

8 **a** The relativistic Doppler effect tells us that light from a source moving away from us is being redshifted, i.e. its wavelength is moved towards the red end of the electromagnetic spectrum. Therefore, the observed wavelength would have been *longer*.

b From Hubble's law:

$v = H_0 d$

where v is km s^{-1} and d is Mpc

$$\therefore H_0 = \frac{v}{d}$$

$$= \frac{1000}{9.55}$$

$= 105\,\text{km s}^{-1}\,\text{Mpc}^{-1}$

Section 2: **Problem solving**

9 **a** Atoms have electrons that are each bound in quantised energy orbitals. When light is incident on the atom, an electron absorbs a photon, which excites the electron to a higher energy state. When the electron returns to any lower energy state, it emits a photon with energy exactly equal to the difference between the energy of the orbitals. Therefore, when observing the light in the emission spectrum of these atoms, the only wavelengths present will be those with energy corresponding to the energy difference of the different atomic orbitals.

b Calculating the energy of this light in electron volts:

$$E = \frac{hc}{\lambda}$$

$$= \frac{4.14 \times 10^{-15} \times 3.00 \times 10^8}{656 \times 10^{-9}}$$

$E = 1.89\,\text{eV}$

This corresponds to the energy transition from $n = 3$ to $n = 2$.

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c Wavelengths of the same length will produce the exact same diffraction pattern, regardless of whether it is a light wave or a matter wave. Hence, calculate the speed at which an electron needs to be travelling in order to have a de Broglie wavelength of 656 nm.

$$\lambda = \frac{h}{p}$$
$$= \frac{h}{mv}$$
$$\therefore v = \frac{h}{m\lambda}$$
$$\therefore v = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 656 \times 10^{-9}}$$

$v = 1.11 \times 10^{3}\,\text{m s}^{-1}$

10 a According to the particle theory, light should have passed directly through the slits to produce two bright lines or bands on the screen. Instead, Young observed a series of bright and dark bands or 'fringes'. Young was able to explain this bright and dark pattern by treating light as a wave. He assumed that the monochromatic light was like plane waves and that, as they passed through the narrow slits, these plane waves were diffracted into coherent (in phase) circular waves. The circular waves would interact, causing interference. The interference pattern produced by these two waves would result in lines of constructive (antinodal) and destructive (nodal) interference that would match the bright and dark fringes respectively.

b For the third dark band, pd = 2.5λ. For the fourth dark band, pd = 3.5λ. That is, the path difference is always one whole wavelength greater for each consecutive dark band. This value has been stated as being equal to 500 nm in this example, therefore λ = 500 nm.

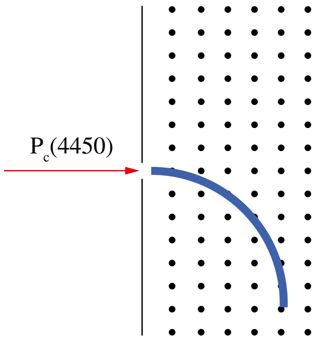
c The covering of one slit still produces an interference pattern; however, it will be no longer uniform. The central band will have the greatest intensity, and each subsequent antinode will decrease in brightness.

d Fringe spacing, Δx, is inversely proportional to the distance between slits. Hence, if the slits are brought closer together then the spacing of the fringes will be further apart.

11 a The total charge of the particle is equal to the sum of the individual charges of each of its constituent quarks. Hence the charge is:

$$\text{u} + \text{u} + \text{d} + \text{c} + \overline{\text{c}}$$
$$\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right) + \left(-\frac{1}{3}\right) + \left(\frac{2}{3}\right) + \left(-\frac{2}{3}\right) = 1$$

b The positively charged particle is subject perpendicularly to a field that is directed out of the page. Therefore, from the right-hand palm rule, the trajectory of this particle will be directed downwards.



c $F = qvB = \frac{mv^2}{r}$

where $m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\therefore r = \frac{mv}{qB}$$
$$= \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}\,qB}$$
$$= \frac{(7.92 \times 10^{-27}) \times 0.999c}{\sqrt{1 - (0.999)^2} \times (1.602 \times 10^{-19}) \times 8.33}$$

$r = 39.8\,\text{m}$

d Quarks interact most strongly via the strong nuclear force; this is what would be binding them together.

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12 a The potential difference provides kinetic energies to the electrons. So the total energy will be this kinetic energy plus the rest mass energy. At high energies such as this, relativistic kinetic energy must be considered. First, convert eV into joules:

$E_k = 90.0 \times 10^3 \times 1.602 \times 10^{-19} = 1.44 \times 10^{-14}$ J

$E_{tot} = E_k + m_0c^2$

$$E_{tot} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E_k + m_0c^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0c^2}{E_k + m_0c^2}$$

$$1-\frac{v^2}{c^2} = \left(\frac{m_0c^2}{E_k + m_0c^2}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{m_0c^2}{E_k + m_0c^2}\right)^2$$

$$\frac{v}{c} = \sqrt{1-\left(\frac{m_0c^2}{E_k + m_0c^2}\right)^2}$$

$$\frac{v}{c} = \sqrt{1-\left(\frac{(9.11\times10^{-31})(3.00\times10^8)^2}{1.44\times10^{-14} + (9.11\times10^{-31})(3.00\times10^8)^2}\right)^2}$$

$v = 0.526c$

$v = 1.578 \times 10^8$ m s^{-1}

b In the first metre the electrons are travelling at 0.999c.

Hence the total energy is:

$$E_{tot} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E_{tot} = \frac{(9.11\times10^{-31})(3.00\times10^8)^2}{\sqrt{1-\frac{(0.9999c)^2}{c^2}}}$$

$$E_{tot} = \frac{(9.11\times10^{-31})(3.00\times10^8)^2}{\sqrt{1-(0.9999)^2}}$$

$E_{tot} = 5.80 \times 10^{-12}$ J

$E_k = E_{tot} - m_0c^2$

$E_k = 5.80 \times 10^{-12} - (9.11 \times 10^{-31})(3.00 \times 10^8)^2$

$E_k = 5.72 \times 10^{-12}$ J

Convert to eV:

$$E_k = \frac{5.72\times10^{-12}}{1.602\times10^{-19}}$$

$E_k = 3.57 \times 10^7$ eV

This is over the first metre. Since we have assumed a linear increase in potential difference, in order to calculate over 10 m, we just multiply by 10:

Potential difference = $3.57 \times 10^7 \times 10$

Potential difference = 357 MeV

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Section 3: **Comprehension**

13 a i The amount that light refracts is wavelength dependent. When white light passes from one material to another and the light waves slow down, the wavelength shortens as the waves bunch up and the wavelengths of each colour change by different amounts. This means that each colour travels at a slightly different speed in the new medium and therefore each colour is refracted by a slightly different amount. Longer wavelengths, such as those of red light, travel the fastest in the new material so they are refracted the least. Shorter wavelengths, such as those in violet light, are slower so they are refracted the most. This means that when light passes through a lens, the angle by which the wavelengths change upon refracting is different, causing the different colours to spread out.

ii Longer wavelengths refract less than shorter wavelengths, hence when a shorter wavelength is refracted, it will spread further than longer wavelengths. You can therefore expect the order to be red, green and then violet—corresponding to their order from longest to shortest wavelength.

b When light passes from a medium of high refractive index to one of low refractive index, it is refracted away from the normal. In this case, as the angle of incidence increases, the angle of refraction gets closer to 90°. Eventually, at an angle of incidence known as the critical angle, the angle of refraction becomes larger than 90° and the light does not undergo refraction; instead it is reflected back into the original medium, as if it were striking a perfect mirror.

From Snell's law: $n_1 \sin\theta_c = n_1 \sin 90°$

$\sin\theta_c = \frac{n_2}{n_1}$

$\sin\theta_c = \frac{1.00}{1.62}$

$\sin\theta_c = 0.617$

$\theta_c = \sin^{-1}(0.617)$

$\theta_c = 38.1°$

c In order for light incident upon a material to create a current, the incident photon must eject an electron. This is a manifestation of the photoelectric effect. Photons will be refracted by the camera lens onto the sensor, at which point an electron is ejected from the surface of the sensor. This photocurrent can then be detected as a signal upon that particular pixel.

d The photoelectric effect has a minimum threshold at which it will no longer eject electrons. Incident light below this threshold frequency will not be energetic enough to be able to stimulate the electrons into being ejected. Radio waves will be well below this threshold frequency, and therefore unable to be detected by the camera's sensor.

e It can be seen from this graph that incident light with wavelength greater than 690 nm is unable to eject electrons. We can infer from this that 690 nm is the threshold wavelength from which we can calculate our work function.

Calculating the energy:

$E_{ph} = \phi + E_k$

$E_{ph} = \phi + 0$

$\phi = \frac{hc}{\lambda}$

$\phi = \frac{(4.14 \times 10^{-15})(3.00 \times 10^{8})}{690 \times 10^{-9}}$

$\phi = 1.80\,\text{eV}$

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